Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

====== [Exercise 3570]

Fermi gas in 2D+3D connected boxes with gravitation

Given a 'mezoscopic' box with $L \times L \times \ell$. L is a macroscopic size while ℓ is a mezoscopic size and s $\ell << L$. We insert N fermions with spin $\frac{1}{2}$ and mass m in to a box. in paragraphs a-d assume the temperature is $T = 0$. (mezoscopic size is very small in relation to a large macroscopic size, in relation to microscopic size).

(a) Describe the uniparticle states density and note what is the energy field which the uniparticle states density in it are like a particle in a two dimensional box.

(b) Find the fermi level E_F in condition it's possible to relate to the box as a two dimensional one. What is the maximum number of N_{max} fermions it's possible to insert in to the box in this condition.

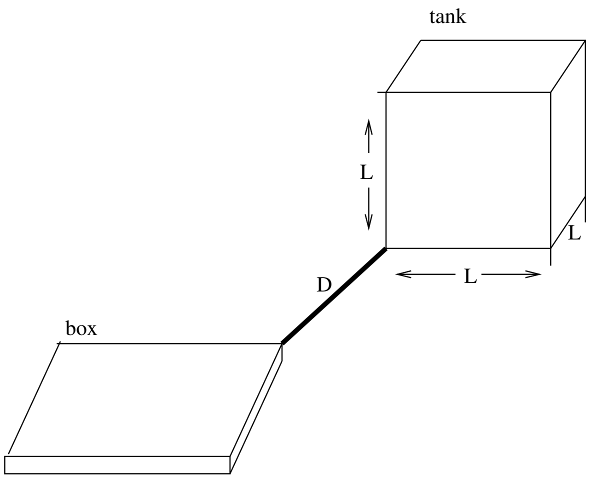
(c) For $N < N_{max}$, as above, find the pressure P on the sided walls of the box, and the force F on the horizontal walls. The box, as above, is attached to a tank with the dimensions $L \times L \times L$ which is D higher from the box. consider gravitation. (assume a very strong gravitation field g, so the question will be reasonable from the order of magnitude).

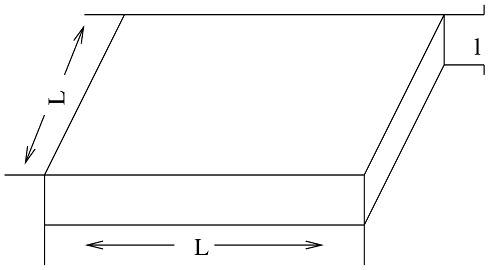
(d) In the conditions of paragraph b', i.e. $N = N_{max}$, What is the minimum hight D_{min} to place the tank so all of the fermions will stay in the box?

(e) The temperature of the system was raised a little bit. Assume temperature T and also $N = N_{max}, D = D_{min}$. as a result, some of the particles that were in the mezoscopic box, transferred to the tank. Estimate the number N' of these particles. For that, assume that the chemical potential of the system is almost not changing as a result of raising the temperature. Express your answers using $T, \mathsf{g}, \mathsf{m}, L, \ell$ only.

given:

$$\int_0^\infty \frac{x^{\frac{1}{2}}}{e^x + 1} dx = \Gamma\left(\frac{3}{2}\right)\left(1 - \frac{1}{\sqrt{2}}\right)\zeta\left(\frac{3}{2}\right) = 0.678$$

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