

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 3570]

Fermi gas in 2D+3D connected boxes with gravitation

Consider a mesoscopic box that has dimensions $L \times L \times \ell$, such that $\ell \ll L$. In the box there are N spin 1/2 electrons. The mass of an electron is m . In items (a-d) assume that the temperature is $T = 0$. In items (d-e) the box is attached to a tank that has dimensions $L \times L \times L$, that is placed at height D relative to the box, and you have to take into account the gravitational field g . Express your answers using m, L, ℓ, g, T .

- Describe the single particle density of states. Specify the energy range over which it is the same as for a two dimensional box.
- Find the fermi energy E_F assuming that it is in the range defined above. What is the maximum number N_{max} of electrons that can be accommodated without violating the 2D description?
- Assuming $N < N_{max}$ find the pressure P on the side walls of the box, and the force F on the horizontal walls.
- Assume $N = N_{max}$. What is the minimum height D_{min} to place the tank such that all of the electrons stay in the box?
- Assume $N = N_{max}$ and $D > D_{min}$. The temperature of the system is raised a little bit. As a result some of the particles that were in the 2D box are transferred to the tank. Estimate their number N' . You are allowed to use any reasonable approximation.



