E3555: Fermions in magnetic field - Landau

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The problem:

Consider N spinless electrons that have mass m and charge e in a $2D$ box that has an area A at zero temperature. perpendicular magnetic field B is applied. The purpose of this question is to find the magnetization of the system.

(1) What are the threshold valueB_ν that is required to empty all the $n > \nu$ Landau levels.
(2) Find the energy$E(B)$ and the magnetization $M(B)$ for strong field $B > B_o$. Give an optional semicalssical derivation to the result assuming that each electron is doing a cyclotron motion with minimal one-particle energy.
(3) Find the energy $E(B)$ and the magnetization $M(B)$ for general $B_\nu < B < B_{\nu+1}$. Explain why the values $E(B_\nu)$ are all equal $E(0)$.
(4) Give a semicalssical derivation to the paramagnetic drops of $M(B)$ at the threshold values B_ν, using the Hall formula for the current along the Edge.

The solution:

(1)

g - degnerate Landau energy levels.

$$g = \frac{eB_\nu A}{hc}. \tag{1}$$

$$g(\nu+1) = N \rightarrow B_\nu = \frac{hc}{eA(\nu+1)}. \tag{2}$$

(2)

$$B > B_o. \tag{3}$$

$$B_o = \frac{Nhc}{eA}. \tag{4}$$

All particles in the lowest Landau level.

$$\frac{E_o}{N} = \frac{\hbar w_o}{2} = \frac{\hbar eB}{2mc}. \tag{5}$$

$$\frac{M}{N} = -\frac{e\hbar}{2mc}. \tag{6}$$

semicalssical derivation

$$\frac{qvB}{c} = \frac{mv^2}{r} \rightarrow E = \frac{e^2B^2r^2}{m2c^2} \tag{7}$$

$$\frac{M}{N} = \frac{IA}{c} \tag{8}$$

$$I = \frac{ev}{2\pi r} = \frac{e^2B}{2\pi mc} \rightarrow \frac{M}{N} = \frac{e^2Br^2}{2mc^2} \tag{9}$$

(3)

$$\epsilon_j = 2\mu_o B(j+\frac{1}{2}), \quad \mu_o = \frac{e\hbar}{2mc}. \tag{10}$$

$$B_o = \frac{nhc}{e}, \quad n = \frac{N}{A} \quad , x = \frac{B}{B_o}. \tag{11}$$

$$x < 1 \rightarrow \quad \frac{1}{j+2} < x < \frac{1}{j+1}. \tag{12}$$

j - lowest landau level that completely filled.

$E = Nx\sum_{j=0}^{j}\epsilon_j + [N-(j+1)Nx]\epsilon_{j+1} =$

$$= \mu_o NBx[(2j+3)-(j+2)(j+1)x]. \tag{13}$$

Magnetization per unit area.

$$M = -\frac{N}{A}\frac{\partial E}{\partial B} = \mu_o n[2(j+1)(j+2)x - (2j+3)]. \tag{14}$$

$E(B_\nu)$ equal to the kinetic energy of the system that only influenced from magnetic field that dont do work on the electron. In the general case $E = V(y) + \hbar w(\nu + \frac{1}{2})$
$V(y)$ depend on the box of area A.

(4)

μ - chemical potential.

$$\mu(\nu) = \frac{B_\nu e\hbar}{mc}(\nu + \frac{1}{2}) \tag{15}$$

$$I_x = R_{xy}V, \quad R_{xy} = \frac{e^2}{h} \tag{16}$$

$$I = \frac{-e}{h}(\mu(\nu) - \mu(\nu-1)) \rightarrow \Delta M = \frac{IA}{c} = \frac{-e^2 A(B_\nu(\nu+\frac{1}{2}) - B_{\nu-1}(\nu-\frac{1}{2}))}{c^2 m}. \tag{17}$$