E3550: Magnetic properties of T=0 electrons (Pauli)

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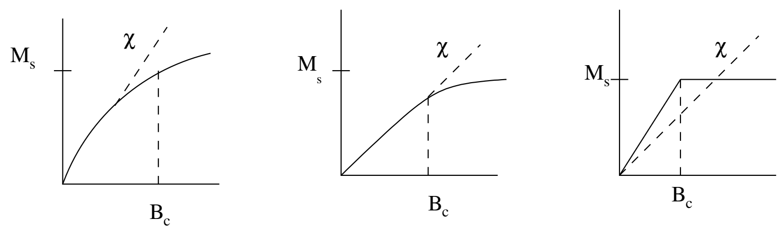
The problem:
N electrons with mass m and spin $\frac{1}{2}$ are placed in a box at zero temperature. A magnetic field B is applied, such that the interaction is $-\gamma B\sigma_z$ where γ is the gyromagnetic ratio. Solve the question for: (I) one-dimensional box with length L.
(II) two-dimensional box with area A.
(III) three dimensional box with volume V.

(a) Describe the single particle density of states function, differ spins up from spins down.

(b) Determine which graph describes the magnetization $M(B)$ of each case (I),(II),(III) and complete the missing details ($M_s=?$,$B_c=?$,$\chi=?$)

Use only γ,m,N,L,A,V.



The solution:

(a) One can easily see that the system can occupy more spin up states.

1D:
For $B<B_c$ there are both spins up and spins down in the system, thus, the total number of spins in an energy level is:

$$N(E)=N_{\downarrow}+N_{\uparrow} \tag{1}$$

$$N(E_f)=4\cdot L(2m)^{1/2}E^{-1/2}=4L(2m)^{1/2}((E_f+\frac{\gamma B}{2})^{-1/2}+(E_f-\frac{\gamma B}{2})^{-1/2}) \tag{2}$$

Magnetization is calculated using the difference between up and down spins:

$$M(B)=\sigma\gamma(N_{\uparrow}-N_{\downarrow})=4L(2m)^{1/2}\frac{\gamma}{2}[(E_f-\frac{\gamma B}{2})^{-1/2}-(E_f+\frac{\gamma B}{2})^{-1/2}]\cong 2L(2m)^{1/2}\frac{\gamma}{2}E_f^{-1/2}(\frac{\gamma B}{2E_f}+\frac{5}{8}(\frac{\gamma B}{2E_f})^3) \tag{3}$$

Calculating the magnetic susceptibility, we use the definition:

$$\chi=\frac{\partial M}{\partial B}=\frac{L}{2}(2m)^{1/2}\gamma^2[(E_f-\frac{\gamma B}{2})^{-3/2}+(E_f+\frac{\gamma B}{2})^{-3/2}] \tag{4}$$

Critical magnetic field B_c is such that all spins are in the up state i.e. low energy level, at $T=0$ it means that $E<E_f$, therefore we can find B_c using the condition:

$$\frac{\gamma B}{2}=E_f\Longrightarrow B_c=\frac{1}{2m}(\frac{\pi N(0)}{8L})^2\frac{2}{\gamma} \tag{5}$$

2D:

$$N(E_f) = 2 \cdot \frac{A\pi}{(2\pi)^2}(2mE) = \frac{2AmE_f}{\pi} \tag{6}$$

Using the same method as before the magnetization is:

$$M(B) = \frac{\gamma}{2}\Delta N = \frac{Am}{2\pi}\gamma^2 B \tag{7}$$

Now calculating the magnetic susceptibility:

$$\chi = \frac{\partial M}{\partial B} = \frac{Am}{2\pi}\gamma^2 \tag{8}$$

the critical magnetic field B_c will be:

$$\frac{\gamma B}{2} = E_f \Longrightarrow B_c = \frac{2\pi N(0)}{Am\gamma} \tag{9}$$

3D: Again we write an expression for the number of spins as:

$$N(E_f) = 2 \cdot \frac{4\pi}{3}\frac{V}{(2\pi)^3} \cdot (2mE)^{3/2} = \frac{V}{3\pi^2}(2m)^{3/2}[(E_f + \frac{\gamma B}{2})^{3/2} + (E_f - \frac{\gamma B}{2})^{3/2}] \tag{10}$$

The magnetization will be:

$$M(B) = \frac{\gamma}{2}\Delta N = \frac{V}{3\pi^2}(2m)^{3/2}\frac{\gamma}{2}[(E_f + \frac{\gamma B}{2})^{3/2} - (E_f - \frac{\gamma B}{2})^{3/2}] \tag{11}$$

$$M(B) = \frac{\gamma}{2}\Delta N \cong \frac{\gamma}{2}\frac{V}{3\pi^2}(2m)^{3/2}\sqrt{E_f}\frac{3}{2} \cdot \gamma B + O(B^3) \tag{12}$$

The Magnetization is therefore:

$$M = \frac{V}{4\pi^2}(2m)^{3/2}\sqrt{E_f} \cdot \gamma^2 B \tag{13}$$

where

$$E_f = \frac{1}{2m}(\frac{3\pi^2 \cdot N(0)}{V})^{2/3} \tag{14}$$

The susceptibility:

$$\chi = \frac{V^{2/3}\gamma^2}{4\pi^2}2m(3\pi^2 N)^{1/3} = const > 0 \tag{15}$$

The critical magnetic field has the same condition as in the 2D case:

$$\frac{\gamma B}{2} = E_f \Longrightarrow B_c = \frac{1}{m\gamma}(\frac{3\pi^2 N(0)}{V})^{2/3} \tag{16}$$

(b)

Using the above calculations we are able to match the right graph to the magnetization functions first graph from the left represent the $1D$ case, the second one refers to the $3D$ case and the third refers to the $2D$ case.