E3540: Ideal Fermi gas in semiconductor

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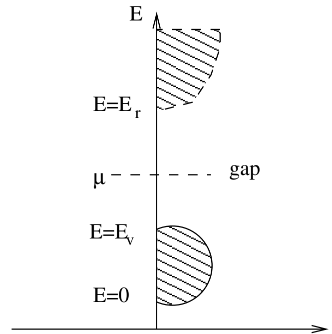
The problem:

Consider a gas of electrons in a metal, coupled to a reservoir at temperature T and chemical potential μ. The single particle density of states is:

$$g(E) = g_v(E) + g_c(E)$$



In the vicinity of the energy gap, we can use the following approximation:

$$g_c(E) \simeq 2\frac{V}{(2\pi)^2} \cdot (2m_c)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}}$$

$$g_v(E) \simeq 2\frac{V}{(2\pi)^2} \cdot (2m_v)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}}$$

Let us define an occupation function for the holes in the valence band:

$$\tilde{f}(E - \mu) \equiv 1 - f(E - \mu)$$

(a) Find the occupation functions of the electrons and the holes in the Boltzmann approximation.
(b) What is the validity of this approximation?
(c) Derive the number of electrons and holes in the conductance and valence band respectively $N_c(\beta, \mu)$ and $N_v(\beta, \mu)$.
(d) Consider a <u>closed</u> system at T=0 the valence band is fully occupied and the conductance band is empty. Now the temperature is raised to T, find the chemical potential and evaluate $N_c(T)$ and $N_v(T)$.

The solution:

(a) Boltzmann's approximation $E_v + T << \mu << E_c - T$ for the occupation function of the electrons in the conductance band:

$$f(E-\mu) = \frac{1}{e^{\beta(E-\mu)} + 1} \simeq e^{-\beta(E-\mu)} \qquad E > E_c \tag{1}$$

and the holes in valence band

$$\tilde{f}(E-\mu) = \frac{1}{e^{-\beta(E-\mu)} + 1} \simeq e^{\beta(E-\mu)} \qquad E < E_v \tag{2}$$

(b) Semiconductors characterised by conductance and valence band levels separated by an energy gap, therefore, the electrons that gains enough energy to cross the gap (leaving a hole in the valence band), makes a very low density of charge carriers that can be treated using Maxwell Boltzmann's statistics.

(c) The number of electrons per unit volume present at temperature T in the conductance band are:

$$n_c(T) = \int_{E_c}^{\infty} dE g_c(E) \frac{1}{e^{\beta(E-\mu)} + 1} \tag{3}$$

The number of holes in the valence band per unit volume is:

$$n_v(T) = \int_{-\infty}^{E_v} dE g_v(E)(1 - \frac{1}{e^{\beta(E-\mu)} + 1}) = \int_{-\infty}^{E_v} dE g_v(E) \frac{1}{e^{-\beta(E-\mu)} + 1} \quad (4)$$

The electrons and holes occupations are:

$$n_c(T) = N_c(T) e^{\beta(\mu - E_c)} \quad (5)$$

$$n_v(T) = N_v(T) e^{\beta(E_v - \mu)} \quad (6)$$

here we added and subtracted E_c to the exponential argument of n_c allowing us to separate a constant term from the integral. We also inserted the given single particle density of sates.

$$N_c(T) = \int_{E_c}^{\infty} dE \cdot \frac{2V}{(2\pi)^2}(2m_c)^{3/2}(E - Ec)^{1/2} exp(-\beta(E - Ec)) \quad (7)$$

taking the constants out and changing variable $x = \beta(E - Ec)$ we stay with:

$$N_c(T) = \frac{2V}{(2\pi)^2}(2m_c)^{3/2} \frac{1}{\beta^{3/2}} \int_{E_c}^{\infty} dx (x)^{1/2} exp(-x) = const \cdot \sqrt{\pi}/2 \quad (8)$$

The integral for the valence bend will be done in a similar way.
Inserting this result in equations (5) and (6) we get the number of charge carriers:

$$N_c(\beta, \mu) = 2 \cdot V \left(\frac{m_c}{2\pi}\right)^{\frac{3}{2}} T^{\frac{3}{2}} exp\left(\frac{\mu - E_c}{T}\right) \quad (9)$$

$$N_v(\beta, \mu) = 2 \cdot V \left(\frac{m_v}{2\pi}\right)^{\frac{3}{2}} T^{\frac{3}{2}} exp\left(-\frac{\mu - E_v}{T}\right) \quad (10)$$

(d) The numbers of charge carriers are not well defined till we have an evaluation of the chemical potential, however we can use the product of equations (5) and (6) to eliminate their dependence on the chemical potential.

$$N_c N_v = 4 \cdot V^2 (\frac{m_c}{2\pi})^{\frac{3}{2}} (\frac{m_v}{2\pi})^{\frac{3}{2}} T^3 exp(-\frac{E_c - E_v}{T}) \quad (11)$$

given that when $T = 0$ the number of charge carriers is zero we therefore deduce that fot $T \neq 0$ $N_c = N_v = N$. assuming $m_c = m_v$:

$$N = 2 \cdot V (\frac{m}{2\pi})^{\frac{3}{2}} T^{\frac{3}{2}} exp(-\frac{E_c - E_v}{2T}) \propto T^{\frac{3}{2}} exp(-\frac{E_g}{2T}) \quad (12)$$

We can now write equation (5) in the following way:

$$n(T) = 2 \cdot (\frac{m}{2\pi})^{\frac{3}{2}} T^{\frac{3}{2}} e^{\beta(\mu - E_c)} \quad (13)$$

equalization of (13) and (12) we are able to evaluate the chemical potential:

$$\mu(T) = \frac{E_c + E_v}{2} \quad (14)$$