Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

====== [Exercise 3540]

Ideal Fermi gas in semiconductor

Given electron gas in a metal. The state uniparticle density function is

$$g(E) = g_v(E) + g_c(E)$$

Assume that near the gap exists area the proximity

$$g_c(E) \simeq L\frac{V}{(2\pi)} \cdot (2m_c)^{\frac{1}{2}} (E - E_c)^{\frac{1}{2}}$$

$$g_v(E) \simeq L\frac{V}{(2\pi)} \cdot (2m_v)^{\frac{1}{2}} (E_v - E)^{\frac{1}{2}}$$

We define an occupation function of the holes in the valence band

$$P^k(E-\mu) \equiv 1 - f(E-\mu)$$

Assume that the metal is attached to the reservoir with temperature β and chemical potential μ. Assume that that $E_v + T << \mu << E_c - T$ and justify the boltzman's approximation in the conductance band $f(E-\mu) \simeq e^{-\beta(E-\mu)}$, and in the valence band $f^k(E-\mu) \simeq e^{\beta(E-\mu)}$. Now, find in the usual way the functions $N_c(\beta,\mu), N_c^h(\beta,\mu)$. The result is

$$N_c(\beta,\mu) = 2\cdot V\left(\frac{m_c}{2\pi}\right)^{\frac{3}{2}} T^{\frac{3}{2}} exp\left(\frac{\mu - E_c}{T}\right)$$

$$N_v(\beta,\mu) = 2\cdot V\left(\frac{m_v}{2\pi}\right)^{\frac{3}{2}} T^{\frac{3}{2}} exp\left(-\frac{\mu - E_v}{T}\right)$$

Now, assume you begin with closed system in temperature zero. It's given that then, the valence band is full and the conductive band is empty. now you raise the temperature (The system is empty!). Assume $m_c = m_v$ and show that

$$\mu(T) = \frac{E_c - E_v}{2}$$

$$N_c = N_v^h \propto T \propto^{\frac{3}{2}} exp\left(-\frac{E_g/2}{T}\right)$$