Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

====== [Exercise 3520]

Ideal Fermi gas in 2D space

Consider N mass m spin 1/2 Fermions, that are are held in a two dimensional box that has an area A. Show that:

$$N(\beta,\mu) \quad = \quad A\frac{\mathsf{m}}{\pi}T\ln\left(1+e^{\frac{\mu}{T}}\right)$$

Tip: Define $X = e^{\beta(E-\mu)}$ and use the integral $\int_1^\infty \frac{dx}{x(x+1)} = \ln\left(\frac{1+X_1}{X_1}\right)$.

Write and explain what is the $T = 0$ result.

Find the chemical potential $\mu(T, N)$.

Find the Fermi energy $E_F \equiv \mu(T \to 0, N)$.

Show that at low temperatures

$$\mu(T) \quad \approx \quad E_F - Te^{-\frac{E_F}{T}}$$

Show that at high temperatures the result is consistent with the Boltzmann approximation.

Find $E(\beta,\mu)$ and $P(\beta,\mu)$ at zero temperature.

Derive the following results:

$$E = A\frac{\pi}{m}\frac{1}{2}\left(\frac{N}{A}\right)^2, \qquad P = \frac{\pi}{m}\frac{1}{2}\left(\frac{N}{A}\right)^2$$

Clarify why at zero temperature $P \propto 1/A^2$, while at high temperatures $P \propto 1/A$.