Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 3510]

State equations for ideal Fermi gas

N fermions with $\frac{1}{2}$ spin and mass m are in a tank with volume V. The gas is in thermic equilibrium in temperature T.
Assume it's possible to relate to the temperature as a low one, and find explicit expressions, up to second order in temperature, for the state equations

$$\mu = \mu\left(T; \frac{N}{V}\right)$$

$$E = E\left(T, V; N\right)$$

$$P = P\left(T, V; N\right)$$

Define what is a low temperature. Use only N, m, V, T. Write expressions also for the heat capacity C_v and the compressibility K_T.

$$K_T \equiv \frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Guideline: Write an expression for $N = N\left(\beta\mu\right)$ and find $\mu\left(\beta, N/V\right)$ while keeping terms up to $O\left(T^2\right)$
Similar to the calculation of $N\left(\beta\mu\right)$ it is possible to calculate $E\left(\beta\mu\right)$ up to second order in temperature.
Now there's to place the expression for $\mu\left(T; \frac{N}{V}\right)$ you found earlier, and write the result as a development of T while keeping terms op to second order only! This is the "trickiest" phase..., You'll have to use the development

$$\left(1+\chi\right)^{\alpha} = 1 + \alpha\chi + \frac{\alpha\left(\alpha-1\right)}{2}\chi^2 + \theta\left(\chi^3\right)$$

several times and to make sure not to losing the first and the second order terms during the algebra process.