Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 3341]

Baruch's B08.

Consider a two dimensional bose gas in a harmonic potential with energy eigenvalues $\hbar\omega(n_1 + n_2 + 1)$ where n_1, n_2 are integers. [This is how the recent cold atom experiments realize condensation].

(a) Show that the average particle number is

$$N = \left(\frac{k_B T}{\hbar\omega}\right)^2 g_2(\zeta) + N_0$$

where N_0 is the the ground state occupation and $g_2(\zeta)$ as defined in class. Assume $k_B T \gg \hbar\omega$ so that summations on n_1, n_2 can be replaced by integrals.

(b) Use $g_2(1) = \pi^2/6$ to infer the Bose Einstein condensation temperature T_c. [Note that N here is not taken to its thermodynamic limit; the transition is still fairly sharp if $N >> 1$.]

(c) Show that for $T < T_c$: $N_0 = N[1 - (T/T_0)^2]$.