Exercises in Statistical Mechanics

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 3341]

Bose in 2D harmonic trap

Consider a two dimensional bose gas in a harmonic potential with energy eigenvalues $(1 + n_1 + n_2)\omega$, where n_1, n_2 are integers. This reflects a conventional setup in actual experiments. Assume that the temperature T is below the Bose-Einstein condensation temperature T_c.

(a) Find the average number $N_e(T)$ of particles in the excited states. Assume $T \gg \omega$ so that summations can be replaced by integrals.

(b) Given that the total number of particles is N what is the Bose-Einstein condensation temperature T_c.

(c) Deduce that the number of condensed particles is $n_0 = N[1 - (T/T_c)^2]$

$$\int_0^\infty \frac{x}{e^x - 1} dx = \frac{\pi^2}{6}$$