Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 3336]

Condesation for general dispersion

An ideal Bose gas consists of particles that ahve the dispersion relation $\epsilon = c|p|^s$ with $s > 0$. The gas is contained in a box that has volume V in d dimentions. The gas is maintained in a uniform temperature T.

(1) Calculate the single particle density of states.

(2) Find a condition involving s and d for the existence of Bose-Einstein condensation. In particular relate to relativistic ($s = 1$) and nonrelativistic ($s = 2$) particles in two dimensions.

(3) Find the dependence of the number of particles N on the chemical potential μ.

(4) Find the dependence of the total energy E on the chemical potential, and show how the pressure P is obtained from this result.

(5) Find an expression for the heat capacity C_v. Show how this result can be expressed using N in the limit of infinite temperature.

(6) Repeat item1 for relativistic gas whose particles have finite mass such that their dispersion relation is $\epsilon = \sqrt{m^2c^4 + c^2p^2}$.

(7) Consider a relativistic gas in $2D$. Find expressions for N and E and P. Should one expect Bose-Einstein condensation?