Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 3338]

Baruch’s B05.

Consider an ideal Bose gas of particles with mass m in a uniform gravitational field of accelaration g.

(a) Show that the critical temperature for the Bose-Einstein condensation is

$$T_c = T_c^0 \left[1 + \frac{8}{9} \frac{1}{\zeta\,(3/2)} \left(\frac{\pi m g L}{k_B T_c^0} \right)^{1/2} \right]$$

where L is the height of the container, $mgL \ll k_B T_c^0$ and $T_c^0 = T_c(g=0)$.
[Hint: $g_{3/2}(\zeta) = g_{3/2}(1) - 2\sqrt{-\pi \ln \zeta} + O(\ln \zeta)$.]

(b) Show that the condensation is accompanied by a discontinuity in the specific heat at T_c,

$$\Delta C_V = -\frac{9}{16\pi} \zeta(3/2) N k_B \left(\frac{\pi m g L}{k_B T_c^0} \right)^{1/2}.$$

[Hint: ΔC_V is due to discontinuity in $(\partial \zeta / \partial T)_{N,V}$].