

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 3042]

Oscillations of a piston in a cylinder filled with gas

Consider a vertically aligned cylinder whose basis has an area A . A piston that has mass M is pushed from above. The piston is held by a spring that has an elastic constant K . If the cylinder is empty the piston is down at zero height ($x = 0$). The cylinder is filled with N gas particles. Each particle has mass m and the temperature is T . Consequently the the piston goes up a distance x , such that the gas occupies a volume Ax . Consider the following 3 cases:

- (a) The temperature is high, such that Boltzmann approximation can be applied.
- (b) The particles are condensed Bosons, T is lower than the condensation temperature.
- (c) The particles are spinless Fermions, and the temperature is zero.

Answer the following questions, relating to each case separately.

1. What is the equilibrium position x_{eq} of the piston?
2. What is the frequency ω of small oscillations?
3. Plot schematic drawing of ω versus T .

Express answers using A , M , K , N , T . The schematic drawing is required to be clearly displayed.