Ex 3030 Bose gas

$$\frac{1}{\lambda} = \left(\frac{mT}{2\pi}\right)^{1/2}$$

$$N = n_0 + \frac{1}{\lambda^3}\left[\Omega_0 \zeta\left(\tfrac{3}{2}\right) + \Omega_V L_{\frac{3}{2}}\left(e^{-\frac{eV}{T}}\right)\right]$$

$$T_c(\infty) = \frac{2\pi}{m}\left(\frac{1}{\zeta\left(\frac{3}{2}\right)}\frac{N}{\Omega_0}\right)^{2/3}$$

$T_c(0)$ = same with $\Omega_0 \mapsto \Omega_0 + \Omega_V$

we assume $T_c(0) \ll T \lesssim T_c(\infty)$

$$eV_c \approx T \ln\left[\frac{\Omega_V}{N\lambda^3 - \Omega_0 \zeta\left(\frac{3}{2}\right)}\right]$$

$$E = \frac{3}{2}\frac{T}{\lambda^3}\Omega_0 \zeta\left(\tfrac{5}{2}\right) + eV \cdot \frac{\Omega_V}{\lambda^3} L_{\frac{3}{2}} + \frac{3}{2}\frac{T}{\lambda^3}\Omega_V L_{\frac{5}{2}}$$

$$C \approx \frac{15}{4}\zeta\left(\tfrac{5}{2}\right)\frac{\Omega_0}{\lambda^3} + \frac{\Omega_V}{\lambda^3}\left(\frac{eV}{T}\right)^2 e^{-\frac{eV}{T}}$$

where we kept leading term in eV/T