Ex3030: Charged Bose gas in box with potential difference

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The problem:

Consider N bozons with mass m, positive charge e and spin 0. The particles are in a tank in thermic equilibrium, and temperature T. The tank has two zones A and B, The volume of each zone is L^3.A battery creates potential difference V between the zones. The potential in every zone is homogenous.

1) Find the condition on N, so if $V = 0$ then there's no condensation, but if $V = \infty$ then there's condensation.
2) assume that the particles in zone A are in a condensation state and the particles in zone B can be described in the Boltzman proximity frame.
   (a) What is the number of the particles in zone B. What is the condition for V, so that Boltzman proximity will be valid
   (b) What is the number of the particles in zone A. How many of them are in condensation state?
   (c) Show that the condensation in zone A as long as $V_c < V$. Find an explicit expression for V_c.

The solution:

1) W e start by calculating that the density of states

$$g(\epsilon) = c \cdot L^3 \epsilon^{\alpha-1} = L^3 \frac{2m^{3/2}}{(2\pi)^2} \epsilon^{1/2} \tag{1}$$

applying this to the term for the number of particles in the system

$$N = \sum_r f(\epsilon_r - \mu) = \int_0^\infty g(\epsilon) \cdot f(\epsilon_r - \mu)\, d\epsilon \tag{2}$$

the explicit solution for this integral is known, by defining the fugacity $z = e^{\beta\mu}$

$$N = \frac{L^3}{\lambda_T^3} Li_{3/2}(z) \tag{3}$$

in order to get a BEC we require that the fugacity goes to unity, then we may use the following $Li_\alpha(1) \sim \zeta(\alpha)$

$$N = n_0 + L^3 \zeta\left(\frac{3}{2}\right) \left(\frac{m}{2\pi}\right)^{3/2} T^{3/2} \tag{4}$$

so we have a lower bound for the numberof particles when $V \to \infty$ in all the excited states of the gas.

now let us find a higher bound when $V = 0$ this can be simply done by rewriting the density of states with the change $L \to 2L$, and from it followes that

$$N = n_0 + 2 \cdot L^3 \zeta\left(\frac{3}{2}\right)\left(\frac{m}{2\pi}\right)^{3/2} T^{3/2} \tag{5}$$

finnaly arriving at

$$L^3 \zeta\left(\frac{3}{2}\right)\left(\frac{m}{2\pi}\right)^{3/2} T^{3/2} < N < 2 \cdot L^3 \zeta\left(\frac{3}{2}\right)\left(\frac{m}{2\pi}\right)^{3/2} T^{3/2} \tag{6}$$

2) (a) We can start from equation (2)

$$N_B = \int_{eV}^{\infty} g(\epsilon) \cdot f(\epsilon_r - \mu)\, d\epsilon \sim \int_{eV}^{\infty} g(\epsilon)\, e^{-\beta(\epsilon_r - \mu)} d\epsilon = \frac{L^3}{\lambda_T^3} e^{-\beta(eV)} \tag{7}$$

from equation (7) we see that, as expected ,the battery effectivly raises the ground state in zone B so now if we want do not wish to create a BEC for which $\mu = eV$, we have to take $T \gg eV$

(b) we are given N particles in the system so $N_A = N - N_B$, in order to know how many particles are condenced we remmember that $N_A = n_0 + L^3 \zeta\left(\frac{3}{2}\right)\left(\frac{m}{2\pi}\right)^{3/2} T^{3/2}$, so the final

$$n_0 = N - N_B - N_e = N - \frac{L^3}{\lambda_T^3}\left[\zeta\left(\frac{3}{2}\right) + e^{-\beta(eV)}\right] \tag{8}$$

(c) in order for there to be a BEC in zone A $n_0 > 0$,starting from Eq (8)

$$N - \frac{L^3}{\lambda_T^3}\left[\zeta\left(\frac{3}{2}\right) + e^{-\beta(eV)}\right] > 0$$

$$\frac{N\lambda_T^3}{L^3} > \zeta\left(\frac{3}{2}\right) + e^{-\beta(eV)}$$

$$eV_c = -T \ln\left[\frac{N\lambda_T^3}{L^3} - \zeta\left(\frac{3}{2}\right)\right] \tag{9}$$

as can see we found a critical voltage, below it there will be no condensate state for the Bosons, that is $V_c < V$ is our condition.