Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 3022]

Bosons with spin is harmonic trap

N Bosons that have spin1 are placed in a 3D harmonic trap. The harmonic trap frequency is Ω. A magnetic field B is applied, such that the interaction is $-\gamma B S_z$, where $S_z = 1, 0, -1$, and γ is the gyromagnetic ratio.

(1) Write an expression for the density of one-particle states $g(\epsilon)$.

(2) Write an expression for the $B = \infty$ condensation temperature T_c.

(3) Write an equation for $T_c(B)$. It should be expressed in terms of the appropriate polylogarithmic funtion.

(4) Find the leading correction in $T_c(B)/T_c \approx 1 + \cdots$ assuming that B is very large. It should be expressed in terms of an elementary function.

(5) Find what is $T_c(B)/T_c$ for $B = 0$, and what is the first-order correction term if B is very small.

(6) Sketch a schematic plot of $T_c(B)/T_c$ versus B. Indicate by solid line the exact dependence, and by dashed and dotted lines the approximations. It should be clear from the figure whether the approximation under-estimates or over-estimates the true result, and what is the B dependence of the slope.

Tips: The prefactors are important in this question. Do not use numerical substitutions.
Use the notation $L_\alpha(z)$ for the polylogarithmic function, and recall that $L_\alpha(1) = \zeta(\alpha)$.
Note also that $L'_\alpha(z) = (1/z)L_{\alpha-1}(z)$, and that $\Gamma(n) = (n-1)!$ for integer n.