BOG (HW 2009 5.2)

Bose gas, Spin 1

$\mathcal{H} = \frac{p^2}{2m} - \gamma B S_z \qquad S_z = \pm 1, 0$

$$n = \sum_p \frac{1}{\frac{1}{z} e^{\beta p^2/2m - \beta\gamma B} - 1} + \sum_p \frac{1}{\frac{1}{z} e^{\beta p^2/2m} - 1} + \sum_p \frac{1}{\frac{1}{z} e^{\beta p^2/2m + \beta\gamma B} - 1}$$

z per σ; $\uparrow S_z = +1$; $\uparrow S_z = 0$; $\uparrow S_z = -1$

$$= \frac{1}{\lambda^3}\left[g_{3/2}(z e^{\beta\gamma B}) + g_{3/2}(z) + g_{3/2}(z e^{-\beta\gamma B})\right]$$

lowest energy state for $S_z = +1$ particles, all terms must be ≥ 0

Condensation at $z e^{\beta\gamma B} = 1 \rightarrow z = e^{-\beta\gamma B}$

at $T < T_c$:

$$n = \frac{1}{\lambda^3}\left[g_{3/2}(1) + g_{3/2}(e^{-\beta\gamma B}) + g_{3/2}(e^{-2\beta\gamma B})\right] + \langle n_0 \rangle$$

$= 0$ if $\gamma B \gg kT$

keep only lowest order correction for large B

for $B = 0 \rightarrow n = \frac{3}{\lambda^3} g_{3/2}(1) = 3 g_{3/2}(1) \cdot \left(\frac{2m\pi kT_c}{h^2}\right)^{3/2}$

$$\rightarrow \boxed{kT_c^0 = \frac{n^{2/3}}{m} \cdot \frac{2\pi\hbar^2}{(3 \cdot 2.612)^{2/3}}}$$

For $\gamma B \gg kT \rightarrow n = \frac{1}{\lambda^3}\left(g_{3/2}(1) + g_{3/2}(e^{-\beta\gamma B})\right) \quad (*)$

$$n \approx \frac{1}{\lambda^3}\left(g_{3/2}(1) + e^{-\beta\gamma B}\right)$$

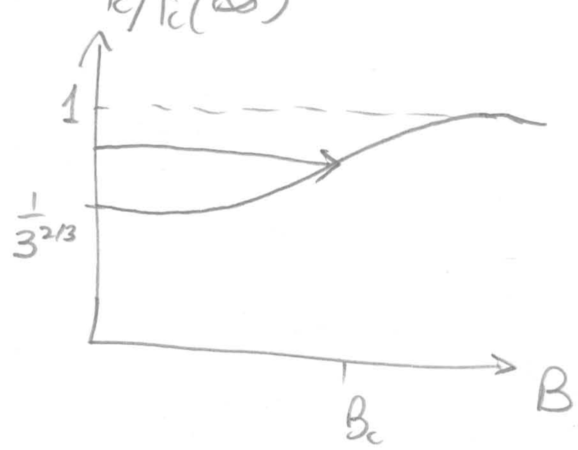
giving

$$\left(\frac{kT_c^{B\to 0}}{kT_c^{B\to\infty}} = \frac{1}{3^{2/3}}\right)$$

$$\boxed{kT_c^{\gamma B \gg kT} = \left(\frac{n}{g_{3/2}(1) + e^{-\beta\gamma B}}\right)^{2/3} \cdot \frac{2\pi\hbar^2}{m} \approx \frac{n^{2/3}}{m} \cdot \frac{2\pi\hbar^2}{(2.612)^{2/3}}}$$

↑ here, use $\boxed{kT_c^0 \approx \frac{2\pi\hbar^2}{(2.612)^{2/3}} \cdot \frac{n^{2/3}}{m}}$

b)



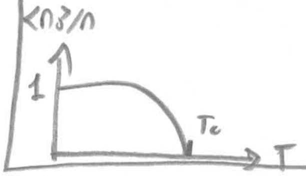
$\frac{\langle n_0 \rangle}{n}(T)$ סעיף ג' החלקים- מושיג דשבה חונק

$$n = \frac{1}{\lambda^3} g_{3/2}(1) + \frac{1}{\lambda^3} e^{-\beta\gamma B} + \langle n_0 \rangle$$

$$\frac{\langle n_0 \rangle}{n} = 1 - \frac{g_{3/2}(1) + e^{-\beta\gamma B}}{n\lambda^3}$$

$\frac{1}{\lambda^3} \sim T^{3/2}$ אזי

לכן



As B is increased, T_c rises until B_c is reached.
at $B = B_c$, $T = T_c$ and condensation occurs. we then have from (*)

$$n\lambda^3 = g_{3/2}(1) + g_{3/2}(e^{-\beta\gamma B_c}) \approx g_{3/2}(1) + e^{-\beta\gamma B_c}$$

$$-\beta\gamma B_c = \ln(n\lambda^3 - g_{3/2}(1))$$

$$\boxed{B_c = \frac{-kT}{\gamma} \ln(n\lambda^3 - g_{3/2}(1))}$$

c) $$E = -\left(\frac{\partial}{\partial \beta} \ln \mathcal{Z}\right)_{z,V} = kT^2 \frac{\partial}{\partial T}\left(\sum_{S_z = 0, \pm 1} \ln(\mathcal{Z}_{S_z})\right) = V k T^2 \sum_{S_z = 0, \pm 1} \frac{\partial}{\partial T}\left(\frac{g_{5/2}(z e^{\beta\gamma B S_z})}{\lambda^3}\right)_{z,V}$$

(Pathria 7.1.11) (Pathria 7.1.1 $\mathcal{Z} = \mathcal{Z}_{S_z=1} \cdot \mathcal{Z}_{S_z=0} \cdot \mathcal{Z}_{S_z=-1}$) Pathria 7.1.7: $\ln \mathcal{Z} = V \cdot \frac{P}{kT} = \frac{V}{\lambda^3} \cdot g_{5/2}(z)$

At $T \leq T_c$: $$\frac{E}{V} = kT^2 \frac{\partial}{\partial T}\left[\frac{g_{5/2}(1)}{\lambda^3} + \frac{g_{5/2}(e^{-\beta\gamma B})}{\lambda^3} + \frac{g_{5/2}(e^{-2\beta\gamma B})}{\lambda^3}\right]$$

$$\approx kT^2 \frac{\partial}{\partial T}\left[\frac{g_{5/2}(1)}{\lambda^3} + \frac{e^{-\beta\gamma B}}{\lambda^3}\right]$$

(קצת אלגברה)

$$= \frac{kT}{\lambda^3} \cdot \frac{3}{2}\left(g_{5/2}(1) + e^{-\beta\gamma B}\right) + \frac{kT}{\lambda^3} \cdot \frac{\gamma B}{kT} e^{-\beta\gamma B}$$

כאשר $\gamma B \gg kT$ כי, נזניח את לכן

$$\approx \frac{kT}{\lambda^3} \cdot \frac{3}{2} g_{5/2}(1) + \frac{\gamma B}{\lambda^3} e^{-\beta\gamma B}$$

$$\frac{C_V}{k_B} = \frac{15}{4} \cdot \frac{1}{\lambda^3} g_{5/2}(1) + \frac{1}{\lambda^3}\left(\frac{\gamma B}{kT}\right)^2 e^{-\beta\gamma B}$$