Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

====== [Exercise 3009]

Entropy and heat capacity of quantum ideal gases

Consider an N particle ideal gas confined in volume V. Find (a) the entropy S and (b) the heat capacity C, highlighting its dependence on the temperature T.

(1) Consider classical gas.

(2) Consider Fermi gas at low temperatures, using leading order Sommerfeld expansion.

(3) Consider Bose gas below the condensation temperature.

(4) Consider Bose gas above the condensation temperature.

(5) What is $C_{Bose}/C_{classical}$ at the condensation temperature?

(6) For temperatures that are above but very close to the condensation temperature, find an approximation for C_V in terms of elementary functions.

Hints: In (4) use the Grand-Canonical formalism to express N and E as a function of the temperature T and the fugacity z. Use the equation for N in order to deduce an expression for $(\partial z/\partial T)_N$. Note that the derivative of the polylogarithmic function $L_\alpha(z)$ is $(1/z)L_{\alpha-1}(z)$. Final results should be expressed in terms of (N, V, T), but it is allowed to define and use the notations λ_T and ϵ_F and T_c. In item (4) the final result can include ratios of polylogarithmic functions, with the fugacity z as an implicit variable. Note that such ratios are all of order unity throughout the whole temperature range provided $\alpha > 1$, while functions with $\alpha < 1$ are singular at $z = 1$.