Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 2360]

The zipper model for DNA molecule

The DNA molecule forms a double stranded helix with hydrogen bonds stabilizing the double helix. Under certain conditions the two strands get separated resulting in a sharp "phase transition" (in the thermodynamic limit). As a model for this unwinding, use the "zipper model" consisting of N parallel links which can be opened from one end. If the links $1, 2, 3, ..., p$ are all open the energy to open to $p+1$ link is ε and if the earlier links are closed the energy to open the link is infinity. The last link $p = N$ cannot be opened. Each open link can assume G orientations corresponding to the rotational freedom about the bond. Construct the canonical partition function. Find then the average number of open links $\langle p \rangle$ as function of $x = Gexp[-\varepsilon/T]$. Plot $\langle p \rangle$ as function of x (assuming N very large). What is the value of x at the transition? Study $\langle p \rangle$ near the transition: what is its slope as $N - - > \infty$? Derive the entropy S. What is it at the transition region and at the transition? Do the same for the heat capacity. What is the order of the transition?