

Exercises in Statistical Mechanics

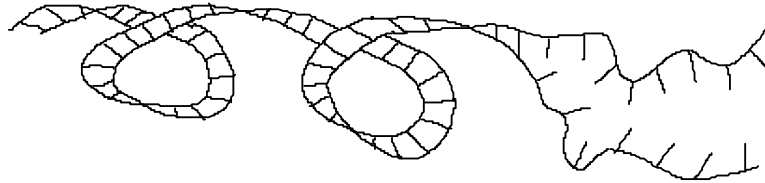
Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

==== [Exercise 2360]

The zipper model for DNA molecule

The DNA molecule forms a double stranded helix with hydrogen bonds stabilizing the double helix. Under certain conditions the two strands get separated resulting in a sharp “phase transition” in the thermodynamic limit. As a model for this unwinding, use the “zipper model” where the DNA is modeled as a polymer with N parallel links that can be opened from one end (see figure).



The energy cost of an open link is ε . A possible state of the DNA is having links $1, 2, 3, \dots, p$ open, and the rest are closed. The last link cannot be opened. Each open link can have g orientations, corresponding to the rotational freedom about the bond. Assume a large number of links N .

- (1) Define $x = ge^{-\varepsilon/T}$ and find the canonical partition function $Z(\beta, x)$.
- (2) Find the average number of open links $\langle p \rangle$ as a function of x .
- (3) Find the linear approximation for $\langle p \rangle$.
- (4) Approximate $\frac{\langle p \rangle}{N}$ for large x .
- (5) Describe the dependence of $\frac{\langle p \rangle}{N}$ on x .
- (6) Find expressions for the entropy $S(x)$ and the heat capacity $C(x)$ at $x = 1$.
- (7) What is the order of the phase transition?