Exercises in Statistical Mechanics

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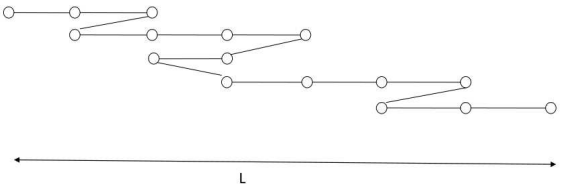
This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 2353]

Tension of a stretched chain

A rubber band is modeled as a single chain of $N \gg 1$ massless non-interacting links, each of fixed length a. Consider a one-dimensional model where the links are restricted to point parallel or anti-parallel to a given axis, while the endpoints are constraint to have a distance $X = (2n - N)a$, where n is an integer. Later you are requested to use approximations that allow to regard X as a continuous variable. Note that the body of the chain may extend beyond the length X, only its endpoints are fixed. In items (c,d) a spring is pushed between the two endpoints, such that the additional potential energy $-KX^2$ favors large X, and the system is released (i.e. X is free to fluctuate).



(a) Calculate the partition function $Z(X)$. Write the exact combinatorial expression. Explain how and why it is related trivially to the entropy $S(X)$.

(b) Calculate the force $f(X)$ that the chain applies on the endpoints. Use the Stirling approximation for the derivatives of the factorials.

(c) Determine the temperature T_c below which the $X = 0$ equilibrium state becomes unstable.

(d) For $T < T_c$ write an equation for the stable equilibrium distance $X(T)$. Find an explicit solution by expanding $f(X)$ in leading order.