A12

2) $Q = \frac{1}{N!}\left(e^{\beta(\chi a - E_\alpha)} + e^{\beta(\chi b - E_\beta)} \right)^N$ — no $1/N!$ since sites distinguishable

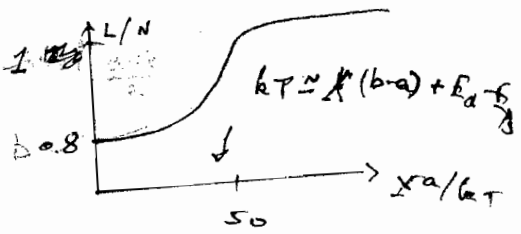
$$L = \frac{1}{\beta}\left(\frac{\partial \ln Q}{\partial \chi}\right)_{T,N} = N\,\frac{a e^{\beta(\chi a - E_\alpha)} + b e^{\beta(\chi b - E_\beta)}}{e^{\beta(\chi a - E_\alpha)} + e^{\beta(\chi b - E_\beta)}} = N\,\frac{a + b\, e^{T_0/T}}{1 + e^{T_0/T}}$$

Keratin molecules in wool: $b/a = 5/4$

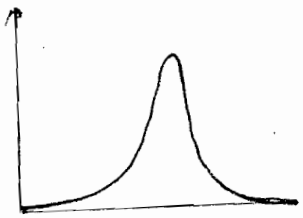
$\frac{E_\beta - E_\alpha}{k_B T} = 10$

$T \to \infty$ only b type if $\chi b - E_\beta > \chi a - E_\alpha$

$T \to \infty$ equal a, b, $L \to N(a+b)/2$



$$\chi_T = \frac{\partial L}{\partial \chi}$$



$$C_\chi = \frac{T}{L}\left(\frac{\partial}{\partial T}\frac{\partial F}{\partial T}\right)_\chi$$

$$S = \frac{\partial F}{\partial T} = -k\beta^2 \frac{\partial -\frac{1}{\beta}\ln Q}{\partial \beta} = k\beta \frac{\partial \ln Q}{\partial \beta} - k \ln Q$$

$$\frac{\partial S}{\partial T} = k^2\beta^2 \left\{ \frac{\partial \ln Q}{\partial \beta} + \beta \frac{\partial^2 \ln Q}{\partial \beta^2} - \frac{\partial \ln Q}{\partial \beta} \right\} = -k^2\beta^2 \frac{\partial^2 \ln Q}{\partial \beta^2}$$

$$\frac{\partial \ln Q}{\partial \beta} = N\,\frac{(\chi a - E_\alpha) e^{\beta(\chi a - E_\alpha)} + (\chi b - E_\beta) e^{\beta(\chi b - E_\beta)}}{e^{\beta(\chi a - E_\alpha)} + e^{\beta(\chi b - E_\beta)}} = \frac{\chi a - E_\alpha + (\chi b - E_\beta) e^{T_0/T}}{1 + e^{T_0/T}}$$

Same structure as above since exponents dominate,

i.e. C_χ has maximum near that of χ_T.