Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 2351]

Tension of a rubber band

The elasticity of a rubber band can be described by a one dimensional model of a polymer. The polymer consists of N monomers that are arranged along a straight line, hence forming a chain. Each unit can be either in a state of length a with energy E_a, or in a state of length b with energy E_b. We define f as the tension, i.e. the force that is applied while holding the polymer in equilibrium.

(1) Write expressions for the partition function $Z_G(\beta, f)$.

(2) For very high temperatures $F_G(T, f) \approx F_G^{(\infty)}(T, f)$, where $F_G^{(\infty)}(T, f)$ is a linear function of T. Write the explicit expression for $F_G^{(\infty)}(T, f)$.

(3) Write the expression for $F_G(T, f) - F_G^{(\infty)}(T, f)$. Hint: this expression is quite simple - within this expression f should appear only once in a linear combination with other parameters.

(4) Derive an expression for the length L of the polymer at thermal equilibrium, given the tension f. Write two separate expressions: one for the infinite temperature result $L(\infty, f)$ and one for the difference $L(T, f) - L(\infty, f)$.

(5) Assuming $E_a = E_b$, write a linear approximation for the function $L(T, f)$ in the limit of weak tension.

(6) Treating L as a continuous variable, find the probability distribution $P(L)$, assuming $E_a = E_b$ and $f = 0$.

(7) Write an expression that relates the function $f(L)$ to the probability distribution $P(L)$. Write also the result that you get from this expression.

(8) Find what would be the results for $Z_G(\beta, f)$ if the monomer could have any length $\in [a, b]$. Assume that the energy of the monomer is independent of its length.

(9) Find what would be the results for $L(T, f)$ in the latter case.

Note: Above a ”linear function” means $y = Ax + B$.
Please express all results using $(N, a, b, E_a, E_b, f, T, L)$.