Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 2311]

Imperfect lattice with defects

A perfect lattice is composed of N atoms on N sites. If n of these atoms are shifted to interstitial sites (i.e. between regular positions) we have an imperfect lattice with n defects. The number of available interstitial sites is M and is of order N. Every atom can be shifted from lattice to any defect site. The energy needed to create a defect is ω. The temperature is T. Define $x \equiv e^{-\omega/T}$.

(a) Write the expression for the partition function $Z(x)$ as a sum over n.
(b) Using Stirling approximation (see note) determine what is the most probable n, and write for it the simplest approximation assuming $x \ll 1$.
(c) Explian why your result for $\bar{n}$ merely reproduces the law of mass action.
(d) Evaluate $Z(x)$ using a Gaussian integral.
(e) Derive the expressions for the entropy and for the specific heat.
(f) What would be the result if instead of Gaussian integration one were taking only the largest term in the sum?

Note: Regarding n as a continuous variable the derivative of $\ln(n!)$ is approximately $\ln(n)$.