Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 2215]

Heat capacity of solids

Consider a piece of solid whose low laying excitations are bosonic modes that have spectral density $g(\omega) = C\omega^{\alpha-1}$ up to a cutoff frequency ω_c, as in the well-known Debye model (items 1-5). Similar description applies for magnetic materials (item 6). In items 7-8 assume that the solid is a ”glass”, whose low laying excitations are like two level entities that have a spectral density $g(\omega)$.

(1) Write a general expression for the energy $E(T)$ of the system. This expression may involve a numerical prefactor that is defined by an α dependent definite integral.

(2) Write a general expression for the heat capacity $C(T)$.

(3) Write a general expression for the variance $\mathrm{Var}(r)$ of an atom that reside inside the solid.

(4) Determine what are α and C and ω_c for a piece of solid that consists of N atoms that occupy a volume L^d in $d = 1, 2, 3$ dimensions, assuming a dispersion relation $\omega = c|k|$, as for ”phonons”.

(5) Write explicitly what are $C(T)$ and $\mathrm{Var}(r)$ for $d = 1, 2, 3$. Be careful with the evaluation of $\mathrm{Var}(r)$. In all cases consider both low temperatures ($T \ll \omega_c$), and high temperatures ($T \gg \omega_c$).

(6) Point out what would be α if the low laying excitations had a dispersion relation $\omega = a|k|^2$ as for ”magnons”.

(7) What is the heat capacity of a ”glass” whose two level entities have excitation energies $\omega = \Delta$, where Δ has a uniform distribution with density C.

(8) What is the heat capacity of a ”glass” whose two level entities have excitation energies $\omega = \omega_c \exp(-\Delta)$, where the barrier $\Delta > 0$ has a uniform distribution with density D.