

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

==== [Exercise 2170]

Polarization of two-spheres system inside a tube

Consider two spheres in a very long hollow tube of length L . The mass of each ball is m , the charge of one ball is $-q$, and the charge of the other one is $+q$. The ball radius is negligible, and the electrostatic attraction between the spheres is also negligible. The spheres are rigid and cannot pass through each other. The spheres are attached by a drop of water. Due to the surface tension there is an attraction force γ that does not depend on the distance. Additionally there is an applied external electric field f . The temperature is T .

- Write the hamiltonian $H(p_1, p_2, x_1, x_2)$ of the system. Rewrite it also in terms of center-of-mass and distance $r = |x_2 - x_1|$ coordinates.
- Calculate the partition function $Z(\beta, f)$ assuming that the drop is not teared out. What is the condition for that?
- Find the probability density function of $\rho(r)$, and calculate the average distance $\langle r \rangle$.
- Find the polarization \tilde{P} as a function of f .
- Expand the polarization up to first order in the field, namely $\tilde{P}(f) = \tilde{P}(0) + \chi f + \mathcal{O}(f^2)$.

Express your answers with L, m, q, γ, T, f .

