Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 2060]

Particle on a ring (rotor) with electric field

Particle with mass m and charge e is free to move along the perimeter of a ring placed on $x-y$ plain, (with a radius R). A uniform, electric field $\bar{\varepsilon}=\hat{x}\varepsilon$ in the system, and it has a thermic balance in temperature T.

(a) Write the Hamiltonian $H\left(p_\theta,\theta\right)$ of the particle.

(b) Calculate the distribution function $Z\left(\beta,\varepsilon\right);\beta^{-1}=k_BT$

(c) What is the probability function $\rho\left(\theta\right)$ of the angled coordinate θ?

(d) Calculate the average place of the particle in a cartesian coordinates (meaning $\langle x\rangle,\langle y\rangle$)

(e) What is the probability function $\rho\left(x\right)$ of the coordinate x? add a schematic drawing.

(f) Express the polarization $P\left(\varepsilon\right)$. For a weak ε develope $P\left(\varepsilon\right)$ up to first order for - ε: $P\left(\varepsilon\right)=p_0+\chi\varepsilon+0\left(\varepsilon^2\right)$ and find χ.

Use the next equations:

$$\frac{1}{2\pi}\int_0^{2\pi}e^{\pm z\ln\theta}d\theta=I_0\left(Z\right)$$

$$I_0^1\left(Z\right)=I_1\left(Z\right)$$

$$I_0\left(Z\right)=1+\frac{1}{4}Z^2+\frac{1}{64}Z^4+...$$