Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 2160]

Particle on a ring with electric field

A particle of mass m and charge e is free to move on a ring of radius R. The ring is located in the (x,y) plan. The position of the particle on the ring is $x = R\cos(\theta)$ and $y = R\sin(\theta)$. There is an electric field $\mathcal{E}$ is the x direction. The temperature is T.

(1) Write the Hamiltonian $H(\theta, p)$ of the particle.

(2) Calculate the partition function $Z(\beta, \mathcal{E})$.

(3) Write an expression for the probability distribution $\rho(\theta)$.

(4) Calculate the mean position $\langle x\rangle$ and $\langle y\rangle$.

(5) Write an expression for the probability distribution $\rho(x)$. Attach a schematic plot.

(6) Write an expression for the polarization. Expand it up to first order in $\mathcal{E}$, and determine the susceptibility.

$$\frac{1}{2\pi}\int_0^{2\pi} \exp(z\cos(\theta))\, d\theta = I_0(z)$$

$$I_0'(z) = I_1(z)$$

$$I_0(z) = 1 + \left(\frac{1}{4}\right) z^2 + \left(\frac{1}{64}\right) z^4 + ...$$