Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 2044]

Boltzmann gas confined in capacitor

An ideal gas is formed of N spinless particles of mass m that are inserted between two parallel plates (Z direction). The horizonatl confinement is due to a two dimensional harmonic potential (XY direction). Accordingly,

$$V(x,y,z) = \begin{cases} \frac{1}{2}\mathsf{m}\omega^2(x^2+y^2) & z_1 < z < z_2 \\ \infty & \text{else} \end{cases}$$

The diatance between the plates is $L = z_2 - z_1$. In the first set of questions (a) note that the partition function Z can be factorized. In the second set of questions (b) an electric field $\mathcal{E}$ is added in the Z direction. Assume that the particles have charge e. Express your answers using $N, \mathsf{m}, L, \omega, e, \mathcal{E}, T$.

(a1) Calculate the classical partition function $Z_1(\beta; L)$ via a phase space integral. Find the heat capacity $C(T)$ of the gas.

(a2) Calculate the quantum partition function for large L. Define what is large L such that the Z motion can be regarded as classical.

(a3) Find the heat capacity $C(T)$ of the gas using the partition function of item (a2). Define what temperature is required to get the classical limit.

(a4) Calculate the forces F_1 and F_2 that the particles apply on the upper and lower plates.

(b1) Write the one-particle Hamiltonian and calculate the classical partition function $Z_1(\beta; z_1, z_2, \mathcal{E})$

(b2) Calculate the forces F_1 and F_2 that are acting on the upper and lower plates. What is the total force on the system? What is the prefactor in $(F_1 - F_2) = \alpha NT/L$.

(b3) Find the polarization $\tilde{\mathcal{P}}$ of the electron gas as a function of the electric field. Recall that the polarization is defined via the formula $\bar{d}W = \tilde{\mathcal{P}}dE$.

(b4) Find the susceptibility by expanding $\mathcal{P}(\mathcal{E}) = (1/L)\tilde{\mathcal{P}} = \chi\mathcal{E} + O(\mathcal{E}^2)$. Determined what is a weak field $\mathcal{E}$ such that the linear approximation is justified.