A25 (2009 3.1)
(exam 2008A.1)

$$\mathcal{H} = \frac{p^2}{2m} \pm \mu B$$

a) $$\lambda_T = \frac{h}{\sqrt{2\pi M k_B T}}$$

$$Z = Z_{id} \times Z_{spin} = \underbrace{\frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N}_{\text{ideal gas}} \cdot \underbrace{\left(2\cosh(\beta\mu B)\right)^N}_{\text{spin system}}$$

$\downarrow$

$$F = F_{kin} + F_{spin}$$

$$S = \left(\frac{\partial F}{\partial T}\right)_{V,N} = S_{kin} + N\frac{\partial}{\partial T}\left(k_B T \ln(2\cosh(\beta\mu B))\right)$$

with $$S_{kin} = N k_B \ln\left(\frac{V}{N\lambda^3}\right) + \frac{5}{2}k_B$$

$$S_{spin} = N k_B \ln(2\cosh(\beta\mu B)) - \beta\mu B \tanh(\beta\mu B)$$

$$\lim_{x\to\infty} \ln(2\cosh x) = \lim_{x\to\infty} \ln(e^x + e^{-x}) = x$$

$$\lim_{x\to\infty} \tanh(x) = 1 \rightarrow \lim_{x\to\infty} x\tanh(x) = x$$

לכן עבור $B\to\infty$ $S_{spin}\to 0$ כצפוי.

ועבור $B\to 0$ $S_{spin} = N k_B \ln 2$

$$S = S_{kin} + S_{spin} = \text{const}$$

תהליך אדיאבטי:

$$\rightarrow S_{kin}(T_i) + S_{spin}(B, T_i) = S_{kin}(T_f) + S_{spin}(0, T_f)$$

$$\rightarrow \boxed{k_B N \ln\left(\frac{\lambda^3_{T_f}}{\lambda^3_{T_i}}\right) = \frac{3}{2}k_B N \ln\frac{T_i}{T_f} = S_{spin}(0, T_f) - S_{spin}(B, T_i)}$$

b) As $B\to\infty$ $S_{spin}(T_i, B) = 0$

$$\rightarrow \ln\frac{T_f}{T_i} = -\frac{2}{3}\ln 2 \rightarrow \boxed{T_f = \frac{T_i}{2^{2/3}}}$$

c) d dimensions: $\lambda^3 \to \lambda^d$, spin has $2S+1$ states

$$\frac{d}{2}\ln\left(\frac{T_f}{T_i}\right) = -\ln(2S+1) \rightarrow T_f = \frac{T_i}{(2S+1)^{2/d}}$$