Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 1816]

Cooling by adiabatic demagnetization

Consider a system of N spins on a lattice at temperature T, each spin has a magnetic moment . In presence of an external magnetic field each spin has two energy levels, μH.

(a) Evaluate the changes in energy δE and in entropy δS as the magnetic field increases from 0 to H. Derive the magnetization $M(H)$ and show that

$$\delta E = T\delta S - \int_0^H M\left(H'\right) dH'.$$

Interpret this result.

(b) Show that the entropy $S\left(E,N\right)$ can be written as $S(M,N)$. Deduce the temperature change when H is reduced to zero in an adiabatic process. Explain how can this operate as a cooling machine to reach $T \approx 10^{-4}K$. (Note: below $10^{-4}K$ in realistic systems spin-electron or spin-spin interactions reduce $S\left(T,H=0\right) \to 0$ as $T \to 0$. This method is known as cooling by adiabatic demagnetization.