

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 1808]

Adiabatic law for generalized dispersion

Consider a gas of noninteracting particles with kinetic energy of the form $\varepsilon(\mathbf{p}) = \alpha|\mathbf{p}|^{3(\gamma-1)}$ where α is a constant; \mathbf{p} is the momentum quantized in a box of size L^3 by $p_x = \hbar n_x/L, p_y = \hbar n_y/L, p_z = \hbar n_z/L$ with n_x, n_y, n_z integers. Examples are nonrelativistic particles with $\gamma = 5/3$ and extreme relativistic particles with $\gamma = 4/3$.

- (a) Use the microcanonical ensemble to show that in an adiabatic process (i.e. constant S, N) $PV^\gamma = \text{const}$.
- (b) Deduce from (a) that the energy is $E = Nk_B T / (\gamma - 1)$ and the entropy is $S = \frac{k_B N}{\gamma - 1} \ln(PV^\gamma) + f(N)$. What is the most general form of the function $f(N)$?
- (c) Show that $C_p/C_v = \gamma$.
- (d) Repeat (a) by using the canonical ensemble.