Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 1040]

Absolute temperature and entropy

In a general quasi static process we defined

$$dQ \equiv dE + dW = \frac{\partial E}{\partial \beta} d\beta + \left(\frac{\partial E}{\partial X} + y \right) dX$$

We expressed E and y by the distribution function $Z(\beta)$
Using differential equations technic, for an integration factor to the non precise differential dQ,you learned in the course, which is solely, a function of β . Show that the integration factor you get is

$$T^{-1}(\beta) = \beta$$

therefore It's possible to write $dQ = TdS$ where dS is a precise differential.
Find the function S and show

$$S = -\frac{\partial}{\partial \left(\frac{1}{\beta}\right)} \left[-\frac{1}{\beta} \ln Z(\beta) \right]$$

By definition, S is an entropy function, and T is called in an absolute temperature.