Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== **[Exercise 1025]**

Fluctuations of N in the grand canonical ensemble

Show that

$$\langle \triangle N^2 \rangle = \left(\frac{1}{\beta} \frac{\partial}{\partial \mu} \right)^2 \ln Z = T \left(\frac{\partial N}{\partial \mu} \right)_{TV}$$

From that prove the equation

$$\frac{\langle \triangle N^2 \rangle}{\langle N \rangle^2} = -\frac{T}{V^2} \left(\frac{\partial V}{\partial p} \right)_{N,T}$$

The last step demands manipulation of equations in thermodynamics
Hints:

$$\left(\frac{\partial N}{\partial \mu} \right)_{T,V} = \left(\frac{\partial \mu}{\partial N} \right)_{TV}^{-1}$$

$$d\mu = \nu dp + S dT, \ \nu = \frac{V}{N}, \ S = \frac{S}{N}$$

$$\frac{\partial \mu}{\partial \nu}|_T = \nu \frac{\partial p}{\partial \nu}|_T$$