Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 0120]

Spectral functions for N particles in a box

In this question one must evaluate $Z(\beta)$ using the next equation

$$Z(\beta) = \sum_n e^{-\beta E_n} = \int g(E)\, d(E)\, e^{-\beta E}$$

(a) Particle in a three dimensional space $H = \sum_{i=1}^{3} \frac{p_i^\alpha}{2m}$
Calculate $g(E)$ and through that evaluate $Z(\beta)$
Guideline: for calculating $\mathcal{N}(E)$ one must evaluate some points $(n_1 n_2 n_3)$- each point represents a state - there's in ellipse $E_{n_1 n_2 n_3} \leq E$

(b) N particles with equal mass in a three dimensional space. assume that it's possible to distinguish between those particles. Prove: $\mathcal{N}(E) = const \cdot E^{\frac{3N}{2}}$
Find the const. use Dirichlet's integral (private case) for calculating the 'volume' of an N dimensional Hyper-ball:

$$\int ... \int \Pi dx_i = \frac{\pi^{\frac{N}{2}}}{(\frac{N}{2})!} R^N$$

$\sum x_i^2 \leq R^2$
Calculate $g(E)$ and from there evaluate $Z(\beta)$