

Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

==== [Exercise 0020]

Average length of a polymer

A polymer can be described as a chain of N monomers. Each monomer has the probability p to be positioned horizontally, adding length a to the polymer, otherwise the monomer adds length b . Let L be the total length of the polymer. Define random variables \hat{X}_n such that:

$$X_n = \begin{cases} a, & \text{the monomer is horizontal} \\ b, & \text{the monomer is vertical} \end{cases}$$

- Express \hat{L} using \hat{X}_n . Using theorems for adding independent random variables find the average length $\langle L \rangle$ and the variance $\text{Var}(L)$.
- Define $f(L) \equiv P(L = na + (N - n)b)$. Find it using combinatorial considerations. Calculate $\langle \hat{L} \rangle$ and $\text{Var}(L)$.
- Define $\sigma_L = \sqrt{\text{Var}(L)}$. What is the behavior of $\sigma_L / \langle L \rangle$ as a function of N ?

