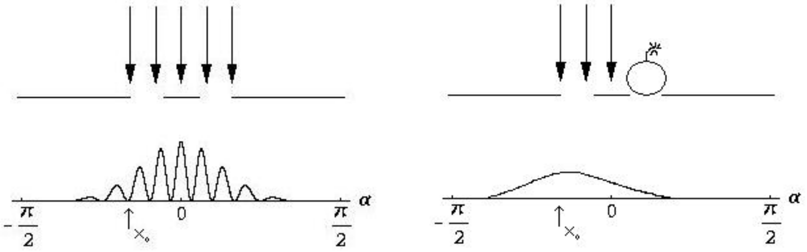
Interaction-Free Measurements

Notes by Ben Yellin

1 Introduction

We are used to think of measurement as an interaction between a object and a measuring device. But there is another way to measure, without interaction: the *Interaction-Free Measurement.* The most simple way to understand IFM is with an example. In our case, the double-slit experiment. Consider a standard two slit measuring apparatus as seen in Figure 1(a). Let x_0 be one of the minima's in the interference pattern. If we place a detector at $x = x_0$, and let the source emit photons one by one, then no photon should reach the detector. Now, consider an object blocking one of the slits (Figure 1(b)). There is no more interference, therefore, there is a possibility for a photon to reach the detector. That's it, we have done an IFM! If a photon has reached the detector, we can be sure that it did not interacted with the object, but if there where no object, there could be no photon reading in the detector. Put it another way: we have "seen" the object, without a photon being scattered from him, we now have a method for *Seeing In The Dark*[1]



(a) With an ordinary setup, a detector placed in x_0 will never observe a photon

(b) A object blocking the right slit can cause a photon to click in the detector

Figure 1: Two slit experiment

This is the essence of IFM, measuring without the entanglement of the observable and the measuring device. The concept was first introduced by Renninger in 1958 in the form of a thought experiment. Consider a α-particle emitting source S, located in the center of two detectors: hemi-

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[1] just another (rather romantic) name for IMF

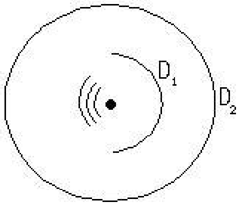


Figure 2: Renninger's thought experiment

spherical detector D_1 with radius R_1 and a spherical detector D_2 with radius R_2, where $R_1 < R_2$ (Figure 2).Let S emit an alpha particle with sharp velocity v at time $t = 0$. The projective measurement operator at time $t_1 = R_1/v$ is

$$U = (I - P^{(1)}) \otimes I + P^{(1)} \otimes D_1 \tag{1}$$

where $P^{(1)}$ is the projector of the measurement of D_1. Note that the first term is not coupled to a detector, this is the trick that let us see in the dark. The probability matrix at time t_1 can be decomposed to

$$\rho \to P^{(1)}\rho P^{(1)} + (I - P^{(1)})\rho(I - P^{(1)}) \tag{2}$$

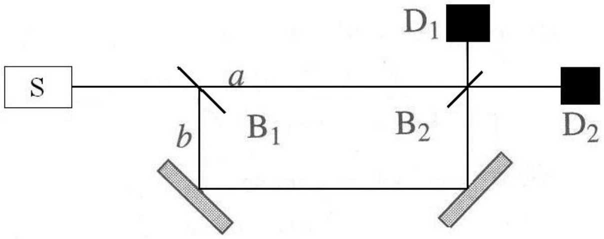
Now, consider the case that at time t_1 detector D_1 hasn't clicked (no detection), then at time $t_1 < t < t_2$ ($t_2 = R_2/v$) the probability matrix is

$$\rho \to (I - P^{(1)})\rho(I - P^{(1)}) \tag{3}$$

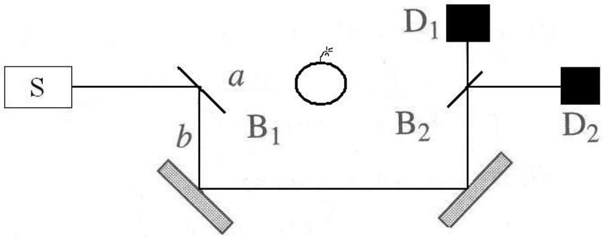
To make my point clear: the wave function collapsed without direct interaction with the measuring device. In case that the outcome of $P^{(1)}$ is zero, than the measuring device has not clicked. Yet from reading D_1 at time $t > t_1$ tells us something about the state of the α-particle.

2 Bomb Testing

The idea of IFM was further developed to a viable detection scheme by Elitzur and Vaidmann. Their idea was to build the Mach-Zender apparatus shown in Figure 3(a). The interferometer is tuned in such way that no photon can hit D_1 due to interference. So, every photon coming from the source, is to be found in detector D_2 with 100% of certainty. To preform a IFM we place an object(from now on - "bomb")[2] on arm a. If we let the source emit a photon, than there is a chance that D_1 will click, so we know that the bomb is there, but without the photon interacting with the bomb (from now on - "the bomb exploded"). The question is, how efficient is that process? Let R and T be the reflection and transmition probabilities of the mirrors, than, the probability of the bomb to explode is T. The probabilities for him to click in detector D_1 (success) or D_2 (unknown) is RT and R^2 respectivly. So, for example, if we use a 50% / 50% mirrors, we have 50% to explode the bomb, 25% for a "useless" detection and only 25% for success. Actually, if we have a click in detector D_2, its not "useless", since we can just send another photon, and than again and again, until detector D_1 click or that the bomb explodes. Therefore, the chance for successful experiment after N repeats is



(a) Without bomb, D_1 wont click



(b) With bomb, 25 % for D_1 to click

Figure 3: Mach-Zender Bomb Tester

$$C_N(R) = RT\sum_{k=1}^{N}(R^2)^{k-1} = RT\left(\frac{1-R^{2N}}{1-R^2}\right) = T\left(\frac{1-R^{2N}}{1+R}\right) \tag{4}$$

where in the last step I used the fact that $T = 1 - R$. Since we can send infinitely many photons (as long as the bomb dis not exploded) the chance of success is given by

$$C_\infty(R) = \lim_{N\to\infty} C_N(R) = \frac{R}{1+R} \tag{5}$$

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[2] Usually the object is called a "bomb" or "photonic nano grains"

Clearly the best result we can obtain will be for $R \to 1$, in that case, the chance for a successful "bomb testing" is 50%.

3 Optimization

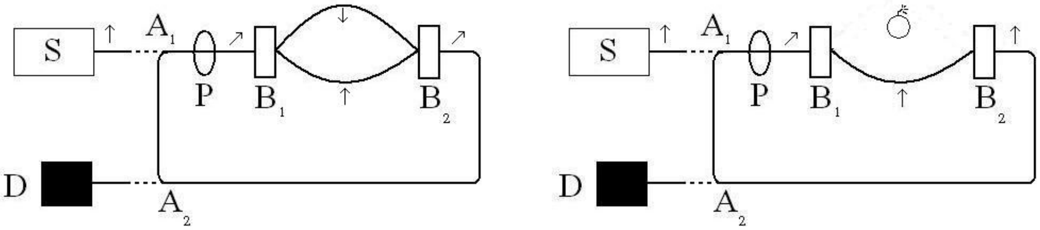
Can we improve that chance? as Obama said: "yes we can". In this section we will see an apparatus that can detect bombs with 100% efficiency (i.e. without exploding them). The experiment apparatus is shown in figure 4(a), It composed from a single spin 1/2 particle emitter S and detector D two "switches" A_1 and A_2 which we can set to choose the path of the particle, a Stern-Gerlach devices B_1 and B_2 and a "spin rotator" P . S emit a particle in the state $|\psi\rangle = |\uparrow\rangle$. Next, the particle pass through P which rotate him in angle α:

$$|\psi\rangle = \cos\left(\frac{\alpha}{2}\right)|\uparrow\rangle + \sin\left(\frac{\alpha}{2}\right)|\downarrow\rangle \tag{6}$$

B_1 split between the up and down components and than B_2 recombine the wave function coherently back to the state it was before entering B_1. Switch A_2 is set to reinject the particle back to the start. After N time passing through A_2 (each time rotated with angle $n\alpha$, $n = 1...N$), we set the switch so that the particle will pass through to the detector. If we set

$$\alpha = \frac{\pi}{N} \tag{7}$$

than the wave function should reach to the detector in the state $|\psi\rangle = |\downarrow\rangle$. Note that the part of B_1 and B_2 could be erased, and the result would remain the same.



(a) Without bomb, the particle rotate in angle α each time it passes through the circle

(b) With bomb,the particle remain in the state $|\uparrow\rangle$

Figure 4: 100% Bomb Tester

Now we place a bomb on arm b (Figure 4(b)). The particle have $\sin^2(\pi/2N)$ chance to hit the bomb. In case the bomb did not exploded, than the particle will continue with the state $|\uparrow\rangle$ and will also emerge from B_2 at that state. Since every time the particle arrive to the the spin rotator, he is in the state $|\uparrow\rangle$, than each time it will have $\sin^2(\pi/2N)$ chance to explode the bomb. But, if the bomb did not exploded(with chance $\cos^2(\pi/2N)$), then the particle is still $|\uparrow\rangle$. Since each time that the particle reach B_1 he have the same probability for not exploding the bomb, then after N repetitions the chance of the particle to reach the detector is given by

$$C(N) = \left(\cos^2\left(\frac{\pi}{2N}\right)\right)^N \tag{8}$$

and for a very large N (small angle) we have $C(N \to \infty) = 1$

Proof: $C(N \to \infty) = 1$

$$\begin{aligned} \ln\left[C(N \to \infty)\right] &= \lim_{N\to\infty} \ln\left[\cos^{2N}\left(\frac{\pi}{2N}\right)\right] = 2\lim_{N\to\infty} \frac{\ln\left[\cos\left(\frac{\pi}{2N}\right)\right]}{\frac{1}{N}} \\ &= 2\lim_{N\to\infty} \frac{-\tan\left(\frac{\pi}{2N}\right)\left(-\frac{\pi}{2N^2}\right)}{-\frac{1}{N^2}} = -\pi \lim_{N\to\infty} \tan\left(\frac{\pi}{2N}\right) = 0 \end{aligned}$$

Therfore

$$C(N \to \infty) = \lim_{N\to\infty} \cos^{2N}\left(\frac{\pi}{2N}\right) = 1 \ \textbf{Q.E.D} \tag{9}$$

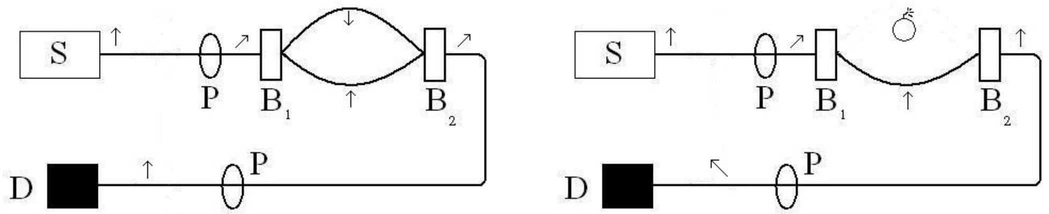
So, if there is a bomb on arm b, than the particle will reach the detector with spin $|\uparrow\rangle$ with certainty, without exploding the bomb. if there is no bomb, than the particle will reach the detector with spin $|\downarrow\rangle$ with certainty. Thats it, we preformed a perfect IFM.

I found it instructive at this point to compare between the Mach-Zender interfrometer from the previous section and the current apparatus. Denoting $\cos(\alpha) = r$ and $r = \sin(\alpha) = t$ Then we can choose the reflection and transmission coeficients for the MZI beam splitters to be $R = r^2$ and $T = t^2$. We set A_2 to lead the beam toward the detctor at the first time (not N times) and place a "spin rotator" $\tilde{P}$, that rotate the spin in angle $-\alpha$, between B_2 and the detector (Figure 5). Without bomb, the particle will reach the detector in the state $|\uparrow\rangle$. with bomb the possible outcome are:

(i) The bomb exploded - T (failure)

(ii) The particle hit the detector with spin up - R^2 (redo experiment)

(iii) The Particle hitted the detector with spin down - TR (success)



(a) Without bomb, the particle rotate in angle α at P than $-\alpha$ at $\tilde{P}$ and reach the detector in the state $|\uparrow\rangle$

(b) With bomb,the particle reach the detector in the state $|\psi\rangle = \cos(\alpha)\,|\uparrow\rangle - \sin(\alpha)\,|\downarrow\rangle$

Figure 5: 100% Bomb Tester used as the MZI

Exactly the same as the previus setup!

Before proceeding to the next chapter, I would like to clarify one last issue. In order to be able to preform the “perfect” bomb test, we have stopped the particle from evolving (we kept him in the state $|\uparrow\rangle$). The concept of keeping the particle in same state, not letting him to evolve in time is known as the *quantom Zeno effect*: By repeatedly (not) measuring a quantom system, we can halt its time evolution. So, (not) measuring the particle causes him to remain in the state $|\uparrow\rangle$. The effect could be explained as follows: The probability for a state to decay into other states is usually given by Fermi Golden Rule. FGR states that the probability to remain in the same state decrease linearly with time, i.e. $P(t) = 1 - \Gamma\delta t$. If we preform N sequential measurments, then after time $t = N\delta t$ (where δt is the interval between each mesurement). The probabilty to remain in the same state is

$$P(t) = (1 - \Gamma\delta t)^N = \left(1 - \frac{\Gamma t}{N}\right)^N \to e^{-\Gamma t} \tag{10}$$

as one could expect. But, FGR is computed under the assumption that the time of the measurement (δt) is much larger than the time needed for the transition. If we preform the measurments in the time of transition, then our probability for transition is $|V\delta t|^2$ (by first order perterbution theory) where V is the perterbution matrix (added to the free hamiltonian). The probabilty to remain in the same state is

$$P(t) = \left(1 - |V\delta t|^2\right)^N = \left(1 - \frac{|V|^2 t^2}{N^2}\right)^N \to 1 \tag{11}$$

so the intial state does not decay into other states.

4 IFM and TSVF

When preforming an IFM, one might ask himself "was the particle gooing through the path with the bomb? can i find a trace for is presence there? (even though the bomb didnt exploded)". Surprisingly, the answer is yes. To do so, we implode the *Two State Vector Formalism* and the idea of weak measurments.

Consider a combination of beam splitters which create a Mach-Zender interferometer nested in another Mach-Zender interferometer (see Figure 6). The internal interferometer is tuned in such a way that a particle coming from the source direction always comes out in detector 3. Now, consider a photon being emitted from the source S and ends up in detector 2. In the framework of TSVF we would say the the photon is pre-selected in the state $|s\rangle$ and post-selected in the state $\langle 2|$. A weak measurment done on arm A will yield a non-zero results, as could be expected. But, a weak measurment done on arm B will also yield results, even though that in out pre and post selected system, the particle wasn't there! To understand that phenomena better, we will inspect the meaning of the weak value measurment for such a proccess.The weak value is given by

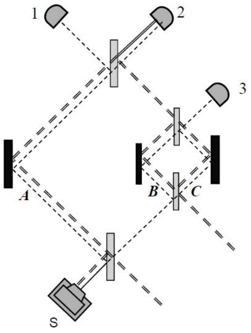


Figure 6: Double nested MZI

$$\langle C_w \rangle_{\Phi,\Psi} = \frac{\langle \Phi| C |\Psi\rangle}{\langle \Phi|\Psi\rangle} \tag{12}$$

in our example, $|\Psi\rangle = U_I |S\rangle$ and $\langle \Phi| = \langle 2| U_{II}$, where U_I is the time evolution operator from time t_i to t_0 and U_{II} is the time evolution from t_0 to t_f (t_0 is the time of the measurement). The total time evolution of the system is given by $U = U_{II}U_I$. Now, if we want to measure a weak value of operator C by placing a (weak) measuring device on arm B, the above expression will take the form of

$$\langle C_w \rangle_{\Phi,\Psi} = \frac{\langle 2|\, U_{II} C U_I \,|S\rangle}{\langle 2|\, U \,|S\rangle} \tag{13}$$

Inspecting the above we see that we can measure weak values at time t_0 in places that the pre-selected state evolved toward them forward in time, represented by $U_I\,|S\rangle$ (the dashed thin line in Figure 6) and the post selected state evolved back in time $\langle 2|\, U_{II}$ (dashed thick line).