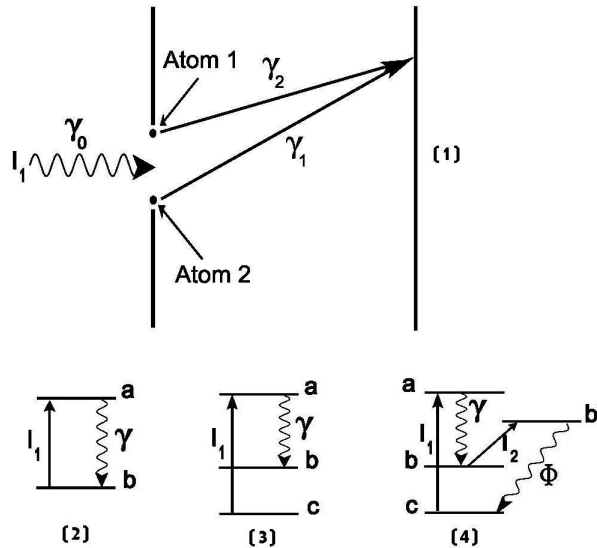


Quantum eraser

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In this lecture we shall see how "erasing information" contained in a system can have an effect on the physical results. The system taken is an analog to the two slit experiment, and the erased information is the "which path" info of the photon, effecting whether or not we receive an interference pattern.

1 An analog to the two slit experiment



Consider a two slit experiment (Fig. 1) in which the slits are replaced by identical 'two level atoms' (Fig. 2), which are resonant with a laser pulse l_1 , impinging the atoms from the left. Assuming only a single photon arrives each time and it has an equal absorption probability in each atom, the state vector of the system after absorption and emission will show:

$$|b, b, \gamma_0\rangle \xrightarrow{l_1, \gamma_0} |b, b, \gamma_1\rangle + |b, b, \gamma_2\rangle$$

Where the first letter represents the energy level of atom 1 (similarly for the second letter), γ_0 is the absorbed laser photon and γ_i is a photon emitted by atom i . Since the system is in a superposition of two paths of the emitted photon and we cannot know which path was taken, we simply get a characteristic young double slit interference pattern.

2 Destroying the interference pattern

Now let us alter the setup and replace the two level atoms with three level atoms as shown in Fig. (3). These atoms are excited to energy level a by pulse l_1 , and decay to b or c emitting a γ_i photon. We set our detection system to detect only the $a \rightarrow b$ radiation transition and ignore the $a \rightarrow c$ radiation. Thus the state vector of the system will evolve as follows:

$$|c, c, \gamma_0\rangle \xrightarrow{l_1, \gamma_0} |b, c, \gamma_1\rangle + |c, b, \gamma_2\rangle$$

This state vector contains information about the path the photon traveled, in the sense that each state of the superposition contains one atom in the excited state b indicating that the photon was

emitted by this atom. The direct result of this state vector is the destruction of the interference pattern. In order to understand this, let's use a relative states notation. First we will look at the two level atoms system state vector:

$$|b, b, \gamma_1\rangle + |b, b, \gamma_2\rangle = (|\gamma_1\rangle + |\gamma_2\rangle) \otimes |b, b\rangle \longrightarrow (|\psi_1\rangle + |\psi_2\rangle) \otimes |\chi^{(1)}\rangle$$

Calculating the expectation value of the projector $P = |\mathbf{r}_0\rangle\langle\mathbf{r}_0|$ will give us the probability of detecting a particale at the detector at \mathbf{r}_0 :

$$\langle P \rangle = \langle \psi | P | \psi \rangle = \langle \psi | \mathbf{r}_0 \rangle \langle \mathbf{r}_0 | \psi \rangle = |\langle \mathbf{r}_0 | \psi_1 \rangle|^2 + |\langle \mathbf{r}_0 | \psi_2 \rangle|^2 + \langle \psi_1 | \mathbf{r}_0 \rangle \langle \mathbf{r}_0 | \psi_2 \rangle + \langle \psi_2 | \mathbf{r}_0 \rangle \langle \mathbf{r}_0 | \psi_1 \rangle$$

Where the cross terms $\langle \psi_1 | \mathbf{r}_0 \rangle \langle \mathbf{r}_0 | \psi_2 \rangle$ and $\langle \psi_2 | \mathbf{r}_0 \rangle \langle \mathbf{r}_0 | \psi_1 \rangle$ are responsible for the interference pattern. Now lets look at the relative state form of the three level atoms system:

$$|b, c, \gamma_1\rangle + |c, b, \gamma_2\rangle \longrightarrow |\psi_1\rangle \otimes |\chi^{(1)}\rangle + |\psi_2\rangle \otimes |\chi^{(2)}\rangle$$

Since the b and c states of the atoms are orthogonal we get $\langle \chi^{(1)} | \chi^{(2)} \rangle = 0$, so the probability of measuring a particle at the detector at \mathbf{r}_0 now gives us:

$$\langle P \rangle = (\langle \psi_1 | \otimes \langle \chi^{(1)} | + \langle \psi_2 | \otimes \langle \chi^{(2)} |) | \mathbf{r}_0 \rangle \langle \mathbf{r}_0 | (|\psi_1\rangle \otimes |\chi^{(1)}\rangle + |\psi_2\rangle \otimes |\chi^{(2)}\rangle) = |\langle \mathbf{r}_0 | \psi_1 \rangle|^2 + |\langle \mathbf{r}_0 | \psi_2 \rangle|^2$$

We see that the cross terms have vanished and consequently with them vanished the interference pattern. We can conclude that the existence alone of the which path information is sufficient to make the interference pattern disappear, regardless of any measurement performed on the system. A simpler way to look at it is through the probability matrix formalism. In the two level atoms case the probability matrix shows: $\rho = p_1\rho_1 + p_2\rho_2 + \rho_{int}$, in which ρ_{int} is responsible for the interference. On the other hand in the 3 level atoms case the probability matrix becomes: $\rho = p_1\rho_1 + p_2\rho_2$, so this is simply the addition of two separate slits, meaning two spots of light on the screen with no interference pattern.

3 Erasing information and unitary evolution

Now, is it possible to "erase" the which path data from the system and reinstate our interference pattern? Let us examine a simple information erasing scheme by modeling our system with qubits. Say we have a 2 slit apparatus in which we can mark the path the particle takes using qubits A and B each marking whether the particle passed through the appropriate slit or not. Another qubit D is the detector. Reading the state of the system with the detector means transferring the "quanta" of data from the marker to the detector, for example reading A means:

$$|A = 1, B = 0, D = 0\rangle \xrightarrow{U_A} |A = 0, B = 0, D = 1\rangle$$

This is done by a unitary evolution of the system using $U_A = e^{-iH_A t}$. Now lets say we prepared the system in the following superposition: $|1, 0, 0\rangle + |0, 1, 0\rangle$, meaning equal probability of each slit, and existence of the which path information in the system. Our purpose is to erase the which path data from the system by using our detector to read the system, namely we want to do the following:

$$|1, 0, 0\rangle + |0, 1, 0\rangle \xrightarrow{U_{A,B}} |0, 0, 1\rangle$$

Where the final state means we know a particle went through (since $D = 1$) but we dont know through which slit because we used $U_{A,B}$. But it is not possible to construct $U_{A,B}$ using H_A and H_B (say according to $U_{A,B} = e^{-i(H_A + H_B)t}$), since a unitary operator acting on two different

orthogonal states must preserve the orthogonality. So $U_{A,B}$ that we are looking for cannot be a unitary operator. In order to get a more intuitive notion of the reason for this, we will further simplify the model by mapping it to a particle (the quanta) in 3 sites system (A, B, D). In this model the hamiltonian H_i couples site i to the D site. So using both of the hamiltonians together will cause the particle's probability to shift from one site to D as desired, but simultaneously cause some 'probability leakage' from D to the other site, since the coupling of the hamiltonians works both ways for each site. So it is not possible to take the particle from the superposition of the A and B sites solely to the D site, and in order to be able to erase the data we will need a more complex erasing scheme.

4 Reinstating the interference pattern

Let us consider the next "information extractor": we replace the three level atoms in our system by four level atoms (Fig. 4). We also apply another laser pulse resonant with the $b \rightarrow b'$ transition, exciting any atom which has emitted a γ photon to the level b' , which is strongly coupled to c by an emission of a ϕ photon. So the state vector of the system will evolve as follows:

$$|c, c, \gamma_0\rangle \xrightarrow{l_1, \gamma} |b, c, \gamma_1\rangle + |c, b, \gamma_2\rangle \xrightarrow{l_2, \phi} |c, c, \gamma_1, \phi_1\rangle + |c, c, \gamma_2, \phi_2\rangle$$

So we may say we have "extracted" the which path information out of the atoms using the radiation field. Now by sending both ϕ photons to a common detector we can reduce the $|\phi\rangle$ states to the vacuum state, creating the following state vector:

$$|c, c, \gamma_1, \phi_1, D = 0\rangle + |c, c, \gamma_2, \phi_2, D = 0\rangle \xrightarrow{\phi_{1,2} \rightarrow \text{detector}} |c, c, \gamma_1, 0, D = 1\rangle + |c, c, \gamma_2, 0, D = 1\rangle$$

We note that the final superposition contains two orthogonal states (since $\langle \gamma_1 | \gamma_2 \rangle = 0$), as required from a unitary evolution. So the introduction of the radiation field allowed us to erase the data in a unitary fashion. The state vector we got is effectively the same as in the two level atoms case, meaning we have erased the record of which atom did the γ photon scattering and reinstated our interference pattern.

5 Delayed choice quantum eraser

If we choose to send only one of the ϕ photons to the detector and block the other one (say using a shutter), then the ϕ detector provides the which path info - for every count on the screen (γ detector) there be will either a count in the ϕ detector signaling the photon was emitted by the free atom or no count in the ϕ detector signaling the photon was emitted by the blocked atom. This scheme allows us to display either the particle (path) or wave (interference) nature of the radiation even if we delay the choice until after the γ photons have been emitted by the atoms.

Sources

M. O. Scully and K. Druhl, *Quantum eraser*

S. P. Walborn, M. O. Terra Cunha, S. Padua, and C. H. Monken, *Double-slit quantum eraser*