# E000: Transmition of a double delta function barrier

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**The problem:**

Calculate the transmition coefficient of two leads connected to an infinite well using the T-Matrix formalism. The hopping amplitude per unit of time from lead number i to the well is $w_i/\sqrt{L_i}$. The density of states in lead number i is $\nu = L_i/(\pi v_f)$. Assume there are no transitions within the well.

**(1)** Calculate the decay constant.
**(2)** Write down the effective hamiltonian of the system.
**(3)** Write down an expression for the resolvent.
**(4)** Write down an expression for the S-Matrix.
**(5)** Write down the transfer coefficient for off-resonant scattering.
**(6)** What happens on ressonance?

**The solution:**
We will be using the following notations: $|k_R\rangle$ and $|k_L\rangle$ to describe the states in the left and right lead respectively and $|k_n\rangle$ to describe the states of the well.

**(1)** We can obtain the decay constant from the "self energy" term:

$$\begin{aligned}\Sigma_{n,m} = \langle n|VG_0V|m\rangle &= \sum_{k_R,k_L} \langle n|V\left[|k_R\rangle\langle k_R| + |k_L\rangle\langle k_L|\right]G_0\left[|k_R\rangle\langle k_R| + |k_L\rangle\langle k_L|\right]V|m\rangle \\ &= \sum_{k_L} \langle n|V|k_L\rangle\langle k_L|G_0|k_L\rangle\langle k_L|V|m\rangle + \sum_{k_R} \langle n|V|k_R\rangle\langle k_R|G_0|k_R\rangle\langle k_R|V|m\rangle \end{aligned} \quad (1)$$

After expanding eq.(1) and pluging in the data given in the question, one gets the following expresion for the decay constant:

$$\Gamma = 2\pi\sum_k |V_k|^2\delta(E-E_k) = \frac{2}{v_F}(w_1^2 + w_2^2) \quad (2)$$

**(2)** The effective hamiltonian is: $H_{eff} = \begin{pmatrix} E_1 & 0 & \dots & 0 \\ 0 & \ddots & & \\ \vdots & & E_n & \\ 0 & & & \ddots \end{pmatrix} + i\Gamma/2 \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{pmatrix}$

**(3)** By definition, the resolvent is: $G(E) = \frac{1}{E-H_{eff}}$

**(4)** The S-Matrix can be found using the Fisher-Lee relation described in L.N. [41.8] : $S_{i,j} = \delta_{i,j} - iT_{i,j}$, where the T-matrix is given by: $T_{i,j} = \langle i|VGV^{\dagger}|j\rangle$. We will calculate the elements of the S-Matrix between the following states (notice the normalization factor):

$$|\phi^{E,i}\rangle = \sqrt{\frac{2L_i}{v_F}}|i=1,2;k_E\rangle \quad (3)$$

$T_{i,j} = \langle i|VGV^{\dagger}|j\rangle = \sum_{k_n} \langle i|V|k_n\rangle\langle k_n|G|k_n\rangle\langle k_n|V^{\dagger}|j\rangle = \frac{2w_i w_j}{v_F} \sum_{n=1}^{\infty} \frac{1}{E-E_n+i\Gamma/2}$

$$S_{i,j} = \delta_{i,j} - i\frac{2w_i w_j}{v_F} \sum_{n=1}^{\infty} \frac{1}{E - E_n + i\Gamma/2} \tag{4}$$

Where $E_n = \frac{1}{2m}(\frac{\pi n}{L})^2$

**(5)** The transmition coefficient is: $T = |S_{1,2}|^2 = (\frac{2w_1 w_2}{v_F})^2 |\sum_{n=1}^{\infty} \frac{1}{E-E_n+i\Gamma/2}|^2$

**(6)** On resonance, only the term $E = E_n$ will contribute. This time we cannot neglect the $i\Gamma/2$ term in the denominator, so on resonance, the transition coefficient is:

$$T = (\frac{2w_1 w_2}{v_F})^2 |\frac{1}{i\Gamma/2}|^2 \tag{5}$$

and after substituting eq.(2) into eq.(6) we get:

$$T = \left|\frac{2w_1 w_2}{w_1^2 + w_2^2}\right| \tag{6}$$

Note that we get total transmition when $w_1 = w_2$. This is the same result as a resonant transmition of a Fabri-Perot interferometer (see for example L.N. [20.9]).