Ex905: Scattering from delta function in perturbation theory

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The problem:

`The correct questions is with |k|<Lambda`

In this exercise we will find an exact expression for the cross-section in scattering from a regularized delta function.
The matrix element of $V(r) = u\delta^3(r)$ between every two momentum states is u for $|k_2 - k_1| < \Lambda$ (otherwise it is zero).

(1) Write the expression for the cross-section in the first-order Born approximation.

(2) Write the expression for the second-order term of T.

(3) Find the exact expression for T by summing up all orders.

The solution:

(1) We will use the relation $\frac{d\sigma}{d\Omega} = |f(\Omega)|^2$. In the first-order case (Born approx.):

$$f(\Omega) = -\frac{m}{2\pi}T_{k_2,k_1} \approx -\frac{m}{2\pi}V_{k_2,k_1} = \begin{cases} -\frac{mu}{2\pi} & |k_2 - k_1| < \Lambda \\ 0 & else \end{cases}$$

$$\frac{d\sigma}{d\Omega} = |f(\Omega)|^2 \approx \frac{m^2u^2}{4\pi^2} \qquad \text{for } |k_2 - k_1| < \Lambda$$

(2) We'll write T up to second order:

$$T_{k_2,k_1} \approx V_{k_2,k_1} + \sum_{k'} V_{k_2,k'}G_{k',k'}V_{k',k_1}$$

The second order term can be written as:

$$\sum_{k'} V_{k_2,k'}G_{k',k'}V_{k',k_1} = \int_0^\Lambda \frac{d^3k}{(2\pi)^3}\left|V_{k,k'}\right|^2 \frac{1}{E - E' + i0} = 2mu^2 \int_0^\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{k_E^2 - k^2 + i0} = 2mu^2\alpha(E)$$

(3) If we look at the 3rd order, we'll see that:

$$T^{[3]} = \sum_{k'}\sum_{k''} VG_{k'}VG_{k''}V = u^3(2m\alpha(E))^2$$

By repeating this process we can find an exact solution for T:

$$T = u\sum_n (2mu\alpha(E))^n = \frac{u}{1 - 2mu\alpha(E)}$$

in the last equation we've used the fact that $2mu\alpha(E) << 1$

we can see that this result is consistent with the result we got in class for the reflection amplitude (lecture notes 41.2):

$$T = \frac{1}{1 - g_E u}V$$