E855: Decay from a-symetric double well

Submitted by: Yevgeni Estrin, Mahary Vasihoun

The problem:

Consider a-symmetric double well, where a single lead is connected to one of the wells.The hopping amplitude between the sites is c and the hopping amplitude to the continuum is σ.
(1) Derive the expression for effective Hamiltonian.
(2) Describe how the eigenvalues of H evolve in the complex plane as γ is increased from 0 to ∞.
(3) Assume the wells are the same energy level and find the eigenstates of the Hamiltonian for:
a)$\gamma >> c$
b)$c >> \gamma$.

The solution:

(1)

$$H = \begin{pmatrix} \epsilon_0 & c & 0 & \cdots & 0 \\ c & \epsilon_1 & \sigma & \cdots & \sigma \\ 0 & \sigma & E_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & \sigma & 0 & \cdots & E_n \end{pmatrix} = \begin{pmatrix} H_0^P & V^{PQ} \\ V^{QP} & H_0^Q \end{pmatrix}$$

$$\Sigma^P = V^{PQ} G_0^Q V^{QP} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \left[\delta(E) - i(\frac{\Gamma}{2}) \right]$$

$$H_{eff} = H_0^P + \Sigma^P = \begin{pmatrix} \epsilon_0 & c \\ c & \epsilon_1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \left[\delta(E) - i(\frac{\Gamma}{2}) \right]$$

(2) Let us find eigenvalues of the Hamiltonian.We will assume that $\delta(E)$ is $\rightarrow 0$ at the continuum.

$$\Gamma = 2\pi\rho(E)\sigma^2 = 2\gamma$$

$$H_{eff} = \begin{pmatrix} \epsilon_0 & c \\ c & \epsilon_1 - i\gamma \end{pmatrix}$$

$$det \begin{vmatrix} \epsilon_0 - \lambda & c \\ c & \epsilon_1 - i\gamma - \lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda_{\pm} = \frac{1}{2} \left[(\epsilon_0 + \epsilon_1 - i\gamma) \pm \sqrt{(\epsilon_0 - \epsilon_1 + i\gamma)^2 + 4c^2} \right]$$

In the case $\epsilon_0 = \epsilon_1 = \epsilon$:

$$\lambda_{\pm} = \frac{1}{2} \left[(2\epsilon - i\gamma) \pm \sqrt{-\gamma^2 + 4c^2} \right]$$

The simplest case of $\epsilon = 0$ is illustrated in Figure 1 and 2. Where the green line represent λ_- and the red λ_+.

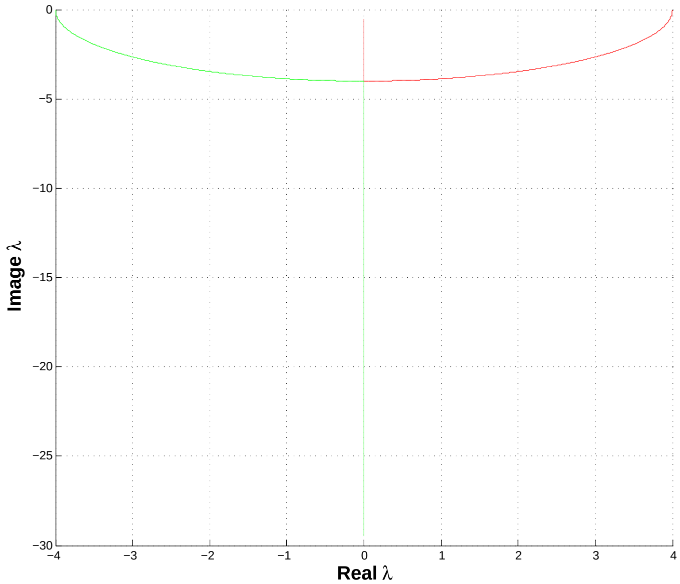

Figure 1: Complex plane of λ

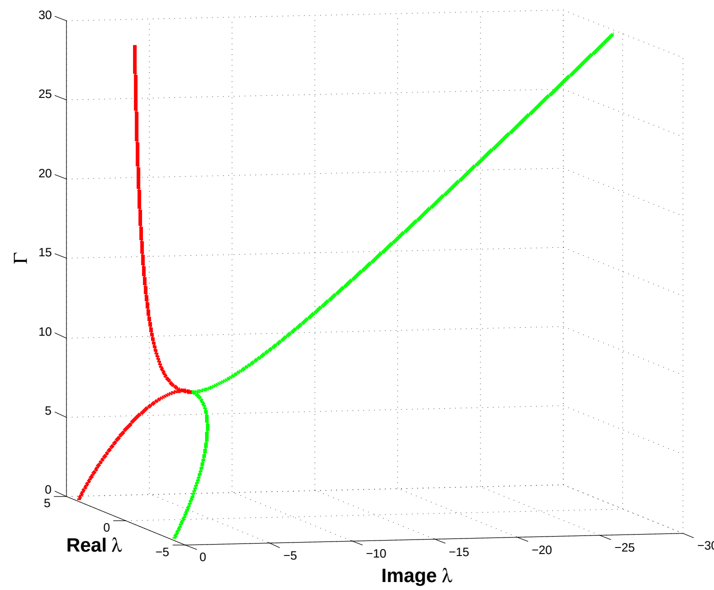

Figure 2: λ as function of Γ

We can see that there is an even decay for the two eigenstates when $c^2 - \gamma^2 > 0$

(3) The eigenstates are:

$$V_{\pm} = \begin{pmatrix} \frac{c}{\sqrt{(\epsilon-\lambda_{\pm})^2+c^2}} \\ \frac{-(\epsilon-\lambda_{\pm})}{\sqrt{(\epsilon-\lambda_{\pm})^2+c^2}} \end{pmatrix}$$

a) $\quad \gamma >> c$

$$\lambda_{\pm} = \frac{1}{2}\left(2\epsilon - i\gamma \pm \sqrt{4c^2 - \gamma^2}\right) \cong \frac{1}{2}\left(2\epsilon - i\gamma \pm i\gamma\right)$$

$$\lambda_{+} \cong \epsilon \quad \Longrightarrow \quad V_{+} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |R\rangle$$

$$\lambda_{-} \cong \epsilon - i\gamma \quad \Longrightarrow \quad V_{-} \rightarrow \begin{pmatrix} \frac{c}{i\gamma} \\ -1 \end{pmatrix} \rightarrow \quad \begin{pmatrix} 0 \\ -1 \end{pmatrix} = |L\rangle$$

b) $\gamma << c$

$$\lambda_\pm = \frac{1}{2}\left(2\epsilon - i\gamma \pm \sqrt{4c^2 - \gamma^2}\right) \cong \frac{1}{2}\left(2\epsilon - i\gamma \pm 2c\right)$$

$$\lambda_+ \cong \epsilon + c \implies V_+ \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\left(|R\rangle - |L\rangle\right) = |A\rangle$$

$$\lambda_- \cong \epsilon - c \implies V_- \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}\left(|R\rangle + |L\rangle\right) = |S\rangle$$