E852: Decay into a continuum from a double well

Submitted by: Boaz Moav

The problem:

Given a particle with mass M.
Length of each arc is L_0. Length of the strait segment is L.
Assume that the energy of the particle is $E \approx E_0 = (\pi/L_0)^2/(2M)$ in the arcs Region.
When the particle escape "out", his velocity is $v_E \approx (2E_0/M)^{1/2}$.
Transmition amplitude per time unit between the arcs is σ_0.
Transmition amplitude per time unit from each of the arcs to the strait section is:
$\sigma = \frac{1}{2} v_E \sqrt{\frac{g}{L_0 L}}$
given $0 < g < 1$
(1) Write the Hamiltonian of the system in the base: $|i = L, R, K\rangle$

The resolvant of the arcs area (2X2 matrix) we will write in this form: $G(z) = \frac{1}{Z - H_{eff}}$

(2) Write the effective Hamiltonian in the expression of the resolvant.
(3) Find the Eigen states of the effective Hamiltonian.
(4) What is the decay constant for each of the Eigen states you have found?

In the second section (2) it is aloud to ignore the Real shift of the energy levels.

The solution:

(1) The Hamiltonian of the system is:

$$\mathcal{H} = \begin{pmatrix} \mathrm{E}_0 & \sigma_0 & \sigma & \sigma & . & . & . & \sigma \\ \sigma_0 & \mathrm{E}_0 & \sigma & \sigma & . & . & . & \sigma \\ \sigma & \sigma & \mathrm{E}_1 & 0 & . & . & . & 0 \\ \sigma & \sigma & 0 & \mathrm{E}_2 & 0 & . & . & 0 \\ . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & 0 \\ \sigma & \sigma & 0 & 0 & 0 & 0 & 0 & \mathrm{E}_k \end{pmatrix} \tag{1}$$

(2) effective Hamiltonian in the expression of the resolvant

The resolvant G(z) is defined in the $P + Q$ formalism as G^p

$$G^p = \frac{1}{Z - H_{eff}} \equiv \frac{1}{Z - (H_0^p + \sum^P)} \tag{2}$$

Where H_0^p is:

$$\mathcal{H}_0^P = \begin{pmatrix} \mathrm{E}_0 & \sigma_0 \\ \sigma_0 & \mathrm{E}_0 \end{pmatrix} \tag{3}$$

and the "self energy" is:

$$\sum^P = V^{PQ} G_0^Q V^{QP} = \sum_k \langle 0|V|k\rangle\langle k|G_0|k\rangle\langle|k|V|0\rangle = \sum_k \frac{|\sigma|^2}{E - E_k + i0} \tag{4}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} (\sum_k \frac{|\sigma|^2}{E - E_k} - i\pi \sum_k |\sigma|^2 \delta(E - E_0)) \equiv \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} (\delta E_0 - i\frac{\Gamma}{2}) \tag{5}$$

The decay constant is derived from the previous equation and is:

$$\Gamma = 2\pi|\sigma|^2 \sum_k \delta(E - E_k) \tag{6}$$

We assume a mean level spacing Δ between the continuum states. The continuum states are states of a one dimentional box with length L and the quantization of the momentum is π/L.
From the relation between the energy and the momentum ($dE = v_E dp$) we get:

$$\Delta = v_E \frac{\pi}{L} \tag{7}$$

Hence the decay constant Receives the form of the Fermi golden rule:

$$\Gamma = 2\pi \frac{1}{\Delta} |\sigma|^2 \tag{8}$$

Hence the effective Hamiltonian is derived by substituting σ and Δ:

$$\mathcal{H}_{eff} = \begin{pmatrix} E_0 & \sigma_0 \\ \sigma_0 & E_0 \end{pmatrix} - i \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{v_E g}{4L_0} \tag{9}$$

(3) The Eigen states of the effective Hamiltonian are symmetric and anti symmetric functions of the arcs system:

$$|A\rangle = \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle) \tag{10}$$

$$|S\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle) \tag{11}$$

(4) The decay constant for the Eigen states can be derived by diagonallyzing the effective Hamiltonian

$$\mathcal{H}_{eff} = \begin{pmatrix} E_0 + \sigma_0 & 0 \\ 0 & E_0 - \sigma_0 \end{pmatrix} - i\Gamma \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{12}$$

we found that for the anti symmetric Eigen state there is no decay ($\Gamma = 0$)

and for the symmetric Eigen state the decay constant is:

$$\Gamma = \frac{v_E g}{2L_0} \tag{13}$$