# E787: Calculation of a cross-section over a "smeared" coulomb target

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**The Problem:**

Write the Born formula for a cross-section given an electrostatic target with charge density $\rho(r)$. Express the result using the **"Form Factor"**, which is defined as the Fourier Transform of charge density.

**The Solution:**

The Born formula for the cross-section, which was derived from Fermi Golden Rule, $\sigma(\Omega)$ is:

$$\sigma(\Omega) = (\frac{M}{2\pi})^2 \cdot \tilde{U}(q) \tag{1}$$

where

$$\tilde{U}(q) = \int U(r)\mathrm{e}^{-iq\cdot r} d^3r \tag{2}$$

$U(r)$ is the scatterer potential. Also, a reminder about where q was originated from:

$$q = k_\Omega - k_0 = 2k_E sin(\theta/2)$$

Where $k_0$ is the original direction of the momentum, $k_\Omega$ is the scattered, $\theta$ is the the angle between them.
Therefore we'll need to solve (2) and get the result by placing (2) in (1). Here our target is "smeared", therefore we cannot assume anything about symmetry in this problem. Any result we'll get will be a general one.

First, our notation for the distances in the problem:

$r\prime$ - the distance from the scattered particle to the "smeared" charge; $r_0$ - the distance between the charge to the volume element of the charge density.
We can write the expression for $U(r^{'})$:

$$U(r\prime) = \int \frac{\rho(r_0)}{|r\prime - r_0|} d^3(r_0)$$

Thus, we can write:

$$\tilde{U}(q) = \int U(r\prime)\mathrm{e}^{-iq\cdot r\prime} d^3(r\prime) = \int \mathrm{e}^{-iq\cdot r'} \int \frac{\rho(r_0)}{|r\prime - r_0|} d^3(r\prime) d^3(r_0) \tag{3}$$

For simplicity let us use $r = r\prime - r_0$. Reorganizing expression (3) gives us:

$$= \int \mathrm{e}^{-iqr} \int \frac{\mathrm{e}^{-iqr_0 \rho(r_0)}}{|r|} d^3(r_0) d^3 r$$

After rearrangement we'll get the final result:

$$\tilde{U}(q) = \int \frac{\mathrm{e}^{-iqr}}{|r|} d^3 r \int \mathrm{e}^{-iqr_0 \rho(r_0)} d^3(r_0) \tag{4}$$

Where:

$$\int \rho(r_0) \mathrm{e}^{-i\vec{q}\cdot\vec{r_0}} d^3(r_0) = \tilde{\rho}(q) \tag{5}$$

$\tilde{\rho}(q)$ is called **"the Form Factor"**. It is simply the Fourier Transform of charge density. By placing (5) and (4) into (1) we get:

$$\sigma(\Omega) = \left( \frac{M}{2\pi} \right)^2 \cdot \tilde{\rho}(q) \int \cdot \frac{\mathrm{e}^{-iqr}}{|r|} d^3 r \tag{6}$$

Where $\tilde{\rho}(q)$ varies according to the symmetry in the problem.

`Remraks:`

`The stages between Eq.(3) and Eq.(4) should be explained.`