E7682: Gamow decay from double well

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The problem:

A particle with a mass m is placed in a box of a length of $2a$. The box is divided into two sections with the same length by a delta function barrier with transmission $g_0 << 1$. The right wall of the box has transmission $g_1 << 1$. The potential floor of all the regions (left section, right section and the region outside the box) is zero. You can assume that the barriers are high and the level splitting is small enough to use Gamow two levels approximation.

The particle is placed in the ground state in the right section of the well.

(1) For $g_1 = 0$ what is the oscillation frequency Ω_0 of the particle between the two sections of the box.
(2) For $g_0 = 0$ what is the decay constant Γ_0 of the particle.

The particle placed in the ground state of the well

(3) What is the decay constant Γ of the particle.

The potential floor in the left section is raised a little, so the potential difference is ϵ

(4) What is the decay constant Γ of the particle.

The solution:

(1) When $g_1 = 0$ it is a double well problem. The momentum state of a ground state is: $p = \frac{\pi}{a}$ and the energy state is: $E = \frac{\pi^2}{2ma^2}$

$$v_E = \frac{dE}{dp}$$

$$v_E = \frac{\pi}{ma}$$

The oscillation frequency between the two sections of the well is:

$$\Omega_0 = \frac{v_E}{ma}\sqrt{g}$$

$$\Omega_0 = \frac{\pi}{ma^2}\sqrt{g_0}$$

(2) When $g_0 = 0$ it is the decay out of a well. The momentum and the energy of the ground state in the zero order remain the same.

$$v_E = \frac{\pi}{ma}$$

The Gamow formula for decay from well:

$$\Gamma_0 = \frac{v_E}{2a}g = \frac{\pi}{2ma^2}g_1$$

(3) The particle in the box can be described as a two level system, with the Hamiltonian:

$$H = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix}$$

$$\vec{\Omega}_0 = (2c, 0, 0)$$

$$\Omega_0 = 2c = \frac{\pi}{2ma}\sqrt{g_0}$$

The probability for the particle to be in the left side of the box is: $P = \frac{1}{2}$ therefor the attempt frequency:

$$\frac{1}{2}\frac{v_E}{2a}$$

$$\Gamma = \frac{1}{2}\Gamma_0$$

(4) In the case that the energy raised in the left cell by ϵ

$$H = \begin{pmatrix} 0 & c \\ c & \epsilon \end{pmatrix}$$

$$\vec{\Omega} = (\frac{\pi}{2ma}\sqrt{g_0}, 0, \epsilon) = (\Omega_0, 0, \epsilon)$$

$$E = \pm\frac{1}{2}\sqrt{(\Omega_0^2 + \epsilon^2)}$$

With eigen states:

$$|+\rangle = \begin{pmatrix} cos(\theta/2) \\ sin(\theta/2) \end{pmatrix}$$

$$|-\rangle = \begin{pmatrix} -sin(\theta/2) \\ cos(\theta/2) \end{pmatrix}$$

When $\theta = arctan(\Omega_0/\epsilon)$.

In the basis:

$$|left\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The probability for the particle to be in the right section:

$$P = |\langle right|-\rangle|^2 = cos^2(\theta/2) = \frac{1}{2}(1 + cos(arctan(\Omega_0/\epsilon) = \frac{1}{2}\left(1 + \frac{\epsilon}{\sqrt{\Omega_0^2 + \epsilon^2}}\right)$$

$$\Gamma_2 = P\Gamma_0$$