E7620:The Effect of periodic driving of wall (Golden rule)

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The problem:

Preparing a particle in high Energy level E_m of a potential well with width L. we move the wall periodically $\triangle L = asin(\omega t)$

(1) Calculate using FGR the rates of transitions Γ to other levels?
(2) Explain the transition probability $P_t(n \mid m)$?
(3) Define the condition on time which the calculation is relevant?

M-mass of the particle.

The solution:

(1)The hamiltonian in the position basis

$$\mathcal{H} = \mathcal{H}_0 + VdL$$

According to Lecture [39.1]

$$[\mathcal{H}_0]_{nm} = \frac{1}{2M}(\frac{\pi}{L}n)^2\delta_{nm}$$

$$V_{nm} = -\frac{\pi^2}{ML^3}nm$$

so

$$\mathcal{H}_{nm} = \frac{1}{2M}(\frac{\pi}{L}n)^2\delta_{nm} - \frac{\pi^2}{ML^3}nm \cdot asin(\omega t) = \mathcal{H}_0 + f(t)W$$

where $f(t) = sin(\omega t) = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$ and $W_{n,m} = -\frac{\pi^2 a}{ML^3}nm$ then according to FGR

$$\Gamma = \frac{2\pi}{\triangle} \mid W_{nm} \mid^2 = 2\pi\varrho(E) \mid W_{nm} \mid^2$$

notice that $\varrho(E)$ is the density of states therfore

$$\varrho(E) = \frac{L}{\pi}\frac{dk}{dE} = \frac{L}{\pi} \cdot \frac{1}{v_E}$$

$$\Gamma = 2\pi\frac{L}{\pi v_E} \mid -\frac{\pi^2 a}{2ML^3}nm \mid^2 = 2\pi\frac{L}{\pi v_E}(\frac{\pi^2 a}{2ML^3}nm)^2$$

in our problem $E_m \gg \omega$

$$\Gamma = 2\pi\frac{L}{\pi v_E} \mid \frac{1}{2}a\frac{1}{ML}(Mv_E)^2 \mid^2$$

because $k_n \sim k_m \sim \sqrt{2ME}$ and $E = \frac{1}{2}Mv_E^2$
so

$$\Gamma = 2\pi \frac{L}{\pi v_E} \times \frac{a^2 M v_E^2}{4L^2} = \frac{a^2 v_E^3 M^2}{2L}$$

(2) The Probability to stay in level m is

$$P(t) = 1 - \sum_{n(\neq m)} P_t(n \mid m) = 1 - \Gamma t = 1 - \frac{a^2 v_E^3 M^2}{2L} \times t$$

the particle can move to energy $E_m + \omega$ we can discribe the distribution of the probability as a sinc function with width $2\pi\hbar/t$ and to $E_m - \omega$ with width $2\pi\hbar/t$.

(3) The condition in which the calculation is valid if t is eual to Heisenberg time

$$t_H = \frac{2\pi\hbar}{\triangle} = \frac{2\pi\hbar}{(\pi/L)v_E}$$

where

$$v_E = \frac{dE_m}{dk} = \frac{d}{dk}(\frac{k_m^2}{2M}) = \frac{k}{M} = \frac{\pi m}{LM}$$