E734: Particle in a one dimension well with pulse step potential

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The problem:

Given a particle in one dimension well with a length L, where $\frac{-x}{2} < L < \frac{x}{2}$.

The given potential is $V(x,t) = \frac{\varepsilon}{1+(\frac{t}{\tau})^2} sgn(x)$

At time $t << -\tau$ the particle is placed on a ground state (n=1).

Use first order of perturbation theory to find probability P(n) of a particle to be at state when $t >> -\tau$.

The solution:

We'll use the following formula of transition probability to get first order perturbation:

$$P_{(n|1)} \approx |W_{n1}|^2 |FT(f_{(t)})|^2 \approx |C_n|^2$$

When

$$C_n = -i \int_0^t e^{t'(E_n - E_1)} V_{n1} dt'$$

Let's define: $w = E_n - E_1$ When $E_n = \frac{\pi^2 n^2}{2ML^2}$ Therefore: $w = \frac{\pi^2 (n^2-1)}{2ML^2}$

Using our given potential

$$V(x,t) = \frac{\varepsilon}{1+(\frac{t}{\tau})^2} sgn(x)$$

We'll calculate $V_{n1}(t)$:

First we'll calculate first order perturbation:

By definition

$$V_{n1}(t) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \psi_n^*(x) \frac{\varepsilon}{1+(\frac{t}{\tau})^2\}} sgn(x) \psi_1(x) dx$$

The known symmetric and anti symmetric solutions for one dimensional well of length L problem is:

For even n: n=0,2,4,. . . $\psi_n(x) = \sqrt{\frac{2}{L}} sin(\frac{2\pi n}{L} x)$

For odd n: n=1,3,5,. . . $\psi_n(x) = \sqrt{\frac{2}{L}} cos(\frac{\pi(2n+1)}{L} x)$

By substituting $\psi_1(x)$ and $\psi_n^*(x)$ in $V_{n1}(t)$ for odd n we get

$V_{n1}(t) = 0$, n=1,3,5,...

Therefore $C_{n1} = 0$ for odd n

And by substituting $\psi_1(x)$ and $\psi_n^*(x)$ in $V_{n1}(t)$ for even n we get

$$V_{n1}(t) = \int_0^{\frac{L}{2}} \frac{L}{2} sin(\frac{2\pi n}{L} x) \frac{\varepsilon}{1+(\frac{t}{\tau})^2} cos(\frac{\pi}{L} x) dx$$

In order to resolve this integral we'll be using the following formula:

$\int_0^\pi sin(nx)cos(mx)dx = \frac{2n}{n^2-m^2}$

And the result will be:

$$\text{V}_{n1}(t) = \frac{8\varepsilon n}{\pi(n^2-1)}\frac{1}{1+(\frac{t}{\tau})^2}$$

, n=0,2,4,...

Now we can calculate the transition amplitude for even n:

$$\text{C}_{n1} = -i\int_0^t e^{iw}V_{n1}dt' = -i\int_0^t e^{iwt'}\frac{8\varepsilon n}{\pi(n^2-1)}\frac{1}{1+(\frac{t}{\tau})^2}dt'$$

In order to calculate this integral we'll use the following solution of FT:

$$\text{F}T\frac{1}{b^2+x^2} = \frac{\pi e^{-bw}}{b}$$

Using this formula and doing some algebra we receive that

$$\text{C}_{n1} = -i\frac{8n\varepsilon\tau}{(n^2-1)}e^{-w\tau}$$

, n=0,2,4,...

Using the first formula we get:

$$\text{P}_{(n|1)} = |C_{n1}|^2 = \frac{64n^2\varepsilon^2\tau^2}{(n^2-1)^2}e^{-\frac{\pi^2(n^2-1)}{ML^2}\tau}$$