# E706: Puls effect

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**The problem:**

In thisroblem we assume that pulse f(t)=f(0)=0. Show that result from problem 702 the same that in problem 704. Note that to prove that problem you shoould assume that energies and matrix elements in adiabatic picture are constant. In other words existing bound (perturbation is weak and slow) that perturbation theory and adiabatic form is valid. So we can get same result from too different ways.

**The solution:**

The Hamiltonian is

$$H = H_0 + V(t)$$

Interaction picture(Perturbation theory):

$$|\psi\rangle = \sum_n c_n(t) e^{-iE_n t} |n\rangle$$

$$Where E_n, |n\rangle$$

Are eigenstates of Hamiltonian:

$$H_0 |n\rangle = E_n |n\rangle$$

(1)

$$i\dot{c}_n = \sum_m e^{i(E_m - E_n)t} V_{nm} c_m$$

$$P(n \to m) = |c_n|^2 = \left| \int_0^t e^{i(E_n - E_m)t} V_{nm} dt \right|^2 + 0(v^4)$$

when:

$$V_{nm}, E_m, E_n$$

Are solved in the base,that Hamiltonian is diagonal. Adiabatic picture:

$$|\psi\rangle = \sum_n a_n(t) |n(t)\rangle$$

$$|n(t)\rangle is eigenstate of H in time t :$$

$$H |n(t)\rangle = E_n(t) |n(t)\rangle$$

(2)

$$\dot{a}_n = -E_n a_n + \sum_m \left\langle \frac{\partial n}{\partial t} \middle| m \right\rangle a_m$$

We want to compare (1) with (2),so we need to define:

$$a_n(t) = b_n(t) e^{-iE_n(t)t}$$

Then

$$\dot{a}_n = \dot{b}_n e^{-iE_n t} + b_n(-i(E_n + \dot{E}_n t))e^{-iE_n t}$$

So equation for b is:

$$\dot{b}_n e^{-iE_n t} - ib_n E_n e^{-iE_n t} - ib_n \dot{E}_n t e^{-iE_n t} = \sum_m \left\langle \frac{\partial n}{\partial t} \,\middle|\, m \right\rangle b_m e^{-iE_m t} - iE_n b_n e^{-iE_n t}$$

$$\dot{b}_n = ib_n \dot{E}_n t + \sum_m \left\langle \frac{\partial n}{\partial t} \,\middle|\, m \right\rangle b_m e^{i(E_n - E_m)t}$$

Now we compare expressions,that for them perturbation is weak and slow:

$$\dot{E}_n \to 0$$

So:

$$\dot{b}_n = \sum_m \left\langle \frac{\partial n}{\partial t} \,\middle|\, m \right\rangle b_m e^{i(E_n - E_m)t}$$

$$P(n \to m) = |a_m|^2 = |b_m|^2 = \left| \int_0^t e^{i(E_n - E_m)t} \left\langle \frac{\partial n}{\partial t} \,\middle|\, m \right\rangle dt \right|^2$$

When:

$$E_m, E_n, n$$

Are in adaibatic picture. Now we will use time-independet perturbation theory to find:

$$n(t_0)$$

$$|n(t_0)\rangle = |n\rangle^{(0)} + \sum_{n \neq m} \frac{V_{nm}(t_0)}{E_n^{(0)} - E_m^{(0)}} |m\rangle^{(0)} \Rightarrow$$

$$\frac{\partial}{\partial t} n \, |n(t_0)\rangle = \sum_{n \neq m} \frac{\dot{V}_{nm}}{E_n^{(0)} - E_m^{(0)}} |m\rangle^{(0)} \Rightarrow$$

$$\left\langle \frac{\partial}{\partial t} n \,\middle|\, m \right\rangle = \frac{\dot{V}_{nm}}{E_n^{(0)} - E_m^{(0)}}$$

$$\underset{adiabatic}{P(n \to m)} = \left| \int_0^t e^{i(E_n^{(0)} - E_m^{(0)})t} \frac{\dot{V}_{nm}}{E_n^{(0)} - E_m^{(0)}} dt \right|^2 \cong$$

$$\cong_{\dot{E}_n = 0} \left| \int_0^t e^{i(E_n^{(0)} - E_m^{(0)})t} \frac{\dot{V}_{nm}}{E_n^{(0)} - E_m^{(0)}} dt \right|^2 =$$

$$= \left| (e^{i(E_n^{(0)} - E_m^{(0)})t} \frac{V_{nm}}{E_n^{(0)} - E_m^{(0)}} )\Big|_0^t - \int_0^t i(E_n^{(0)} - E_m^{(0)}) e^{i(E_n^{(0)} - E_m^{(0)})t} \frac{V_{nm}}{(E_n^{(0)} - E_m^{(0)})} dt \right|^2 =$$

$$= \left| \int_0^t e^{i(E_n^{(0)} - E_m^{(0)})t} V_{nm} dt \right|^2$$

So:

$$\underset{adiabatic}{P(n|\, m)} = \underset{\text{interaction}}{P(n|\, m)}$$