# E436: The connection between the J=1 representation and the Euclidian representation

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**The problem:**

(1) Write the generators of the rotation group in the standard representation J=1.
(2) Define the linear polarization states $|e_x\rangle$, $|e_y\rangle$, $|e_z\rangle$ as the new basis for the representation.
(3)Show that the generators of the new base consolidate with the Euclidian representation.

**The solution:**

(1) The generators of the rotation group in the standard representation J=1 are:

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad S_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(2) We define

$$|\vec{e_z}\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

In oredr to recive $|e_x\rangle$, $|e_y\rangle$ we'll rotate $|e_z\rangle$ once around the y-axis by 90 degree and once around the x-axis by 90 degree:

$$|\vec{e_x}\rangle = U(90, \vec{e_y})|\vec{e_z}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \qquad |\vec{e_y}\rangle = U(90, \vec{e_x})|\vec{e_z}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 0 \\ i \end{pmatrix}$$

(3) In order to show that the generators of the new base consolidate with the Euclidian representation we will show there's a transformation from the Euclidian base to the standard base. We define the matrix T as:

$$T = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \end{pmatrix}$$

and the inverse matrix $T^{-1}$ as:

$$T^{-1} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$$

Using this 2 matrices on $S_x$ we recive:

$$T^{-1}S_xT = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} = M_x$$

in the same way we will see that:

$$T^{-1}S_yT = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} = M_y$$

$$T^{-1}S_zT = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = M_z$$

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