E434: Polarization states "polar" and "linear" of spin 1 particales

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The problem:

A circular polarization state

$$|circular\theta\phi\rangle$$

is defined in the following way: rotation of the state $m = 1$ by the angle θ round the y axis and then a rotation by the anglw ϕ round the z axis.

A linear polarization state

$$|linear\theta\phi\rangle$$

is defined in the following way: rotation of the state $m = 0$ by the angle θ round the y axis and then a rotation by the anglw ϕ round the z axis.

The solution:

(*) The circular state is represented by the vector:

$$|m = 1\rangle = |\Uparrow\rangle \mapsto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

In order to rotate the states we will use the following two matrices, the first rotate round the y axis and the secound round the z axis:

$$U(\vec{e_y}\theta) = \begin{pmatrix} \frac{1}{2}(1+\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ \frac{1}{\sqrt{2}}\sin\theta & \cos\theta & -\frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{pmatrix}$$

$$U(\vec{e_z}\phi) = \begin{pmatrix} \mathrm{e}^{-i\phi} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{i\phi} \end{pmatrix}$$

After writing this two matrices we will use them in order to rotate the $m = 1$ vector state:

$$|\vec{e}\theta\phi\rangle = U(\vec{e_z}\phi)U(\vec{e_y}\theta)|\Uparrow\rangle = \begin{pmatrix} \frac{1}{2}(1+\cos\theta)\mathrm{e}^{-i\phi} \\ \frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta)\mathrm{e}^{i\phi} \end{pmatrix}$$

(**)We will use the exact same procedure to deal with the linear polarization state $m = 0$ and the vector state that goes with it is:

$$|m = 0\rangle = |\Updownarrow\rangle \mapsto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

and we will get

$$|\vec{e}\theta\phi\rangle = U(\vec{e_z}\phi)U(\vec{e_y}\theta)|\Updownarrow\rangle = \left(\begin{array}{c} -\frac{1}{\sqrt{2}}\sin\theta \mathrm{e}^{-i\phi} \\ \cos\theta \\ \frac{1}{\sqrt{2}}\sin\theta \mathrm{e}^{i\phi} \end{array} \right)$$