E363: Electrons in finite potential well + Magnetic field

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The problem:

There's given a two-dimensional potential well $L * L$. The well has depth $V(x) = -V_0$,while outside $V(x) = 0$. The well is shielded by very high walls, and therefore the eigenstates inside the well are like those of "infinite potential well". In the consequence of this all electrons with energy $E > 0$ emerge out from the well. Assume that remained electrons in the box have mass m, charge e and have no spin.

(1) Write a condition $L_{min} < L$ so that at least one electron could be located in the well.
- Assume below that $L_{min} \ll L$.

(2) Calculate number of electrons N_0 that could be located in the well.
- A homogenic magnetic field B is created perpendicular to the plane where the potential well is stuated.
- In the next sections can be used known results about Landau levels (without any proof).

(3) Write a condition $B < B_{max}$ so that electrons could be inside the potential well.
(4) Calculate number of electrons N_1 that could be inside the well, when $B \to \ B_{max}$.
(5) Find the ratio $\frac{N_1}{N_0}$.

The solution:

(1)

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$$

$$E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2m}(\frac{n_x^2}{L^2} + \frac{n_y^2}{L^2}) - V_0.$$

The condition that at least one electron is in the well implies:

$$E_{1,1} < 0$$

$$\frac{\hbar^2\pi^2}{2m}(\frac{2}{L^2}) - V_0 < 0$$

$$L^2 > \frac{\hbar^2\pi^2}{mV_0} = L^2_{min}$$

(2) Number of electrons is number of energy levels in the potential well $\frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2) - V_0 \leq 0$. We count how many n_x, n_y are when $(n_x^2 + n_y^2)_{max} = \frac{2mL^2V_0^2}{\hbar^2\pi^2}$:

$$N_0 = \frac{\pi}{4}(n_x^2 + n_y^2)$$

$$N_0 = \frac{\pi}{4}(\frac{2mL^2V_0}{\hbar^2\pi^2}) = \frac{mL^2V_0}{2\hbar^2\pi}$$

(3) The minimal Landau energy level is $E = \frac{1}{2}\hbar\omega_0$ so we demand:

$$\frac{1}{2}\hbar\omega_0 - V_0 < 0$$

$$\frac{e\hbar B}{2mc} < V_0$$

$$B < \frac{2mcV_0}{e\hbar} = B_{max}$$

(4) We are interested in variation of k_x from wich we get the Landau expression for g:

$$g = \frac{eBL^2}{2\pi\hbar c}$$

$$N_1 = \frac{eB_{max}L^2}{\pi\hbar c} = \frac{mV_0L^2}{\pi\hbar^2}$$

(5) From sections (2) and (4) we get:

$$\frac{N_1}{N_0} = 2.$$